Chapter 5

Analyzing Linear Equations

What You'll Learn

- **Lesson 5-1** Find the slope of a line.
- **Lesson 5-2** Write direct variation equations.
- **Lessons 5-3 through 5-5** Write linear equations in slope-intercept and point-slope forms.
- **Lesson 5-6** Write equations for parallel and perpendicular lines.
- **Lesson 5-7** Draw a scatter plot and write the equations of a line of fit.

Key Vocabulary

- **slope** (p. 256)
- **rate of change** (p. 258)
- **direct variation** (p. 264)
- **slope-intercept form** (p. 272)
- **point-slope form** (p. 286)

Why It's Important

Linear equations are used to model a variety of real-world situations. The concept of slope allows you to analyze how a quantity changes over time.

You can use a linear equation to model the cost of the space program. The United States began its exploration of space in January, 1958, when it launched its first satellite into orbit. In the 1970s, NASA developed the space shuttle to reduce costs by inventing the first reusable spacecraft.

You will use a linear equation to model the cost of the space program in Lesson 5-7.

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Getting Started

▶ **Prerequisite Skills** To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 5.

For Lesson 5-1   **Simplify Fractions**

Simplify. *(For review, see pages 798 and 799.)*

1. $\dfrac{2}{10}$ **2.** $\dfrac{8}{12}$ **3.** $\dfrac{2}{-8}$ **4.** $\dfrac{-4}{8}$

5. $\dfrac{-5}{-15}$ **6.** $\dfrac{-7}{-28}$ **7.** $\dfrac{9}{3}$ **8.** $\dfrac{18}{12}$

For Lesson 5-2   **Evaluate Expressions**

Evaluate $\dfrac{a-b}{c-d}$ for each set of values. *(For review, see Lesson 1-2.)*

9. $a = 6, b = 5, c = 8, d = 4$ **10.** $a = 5, b = -1, c = 2, d = -1$

11. $a = -2, b = 1, c = 4, d = 0$ **12.** $a = 8, b = -2, c = -1, d = 1$

13. $a = -3, b = -3, c = 4, d = 7$ **14.** $a = \dfrac{1}{2}, b = \dfrac{3}{2}, c = 7, d = 9$
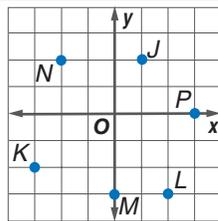
For Lessons 5-3 through 5-7   **Identify Points on a Coordinate Plane**

Write the ordered pair for each point.
(For review, see Lesson 4-1.)

15. J **16.** K

17. L **18.** M

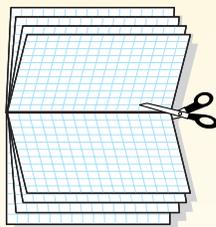**19.** N **20.** P

FOLDABLES™
Study Organizer

Writing Linear Equations Make this Foldable to help you organize your notes. Begin with four sheets of grid paper.
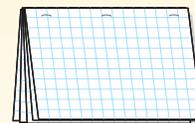
Step 1 **Fold and Cut**

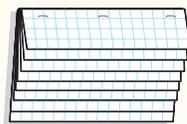Fold each sheet of grid paper in half along the width. Then cut along the crease.

Step 2 **Staple**

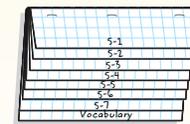Staple the eight half-sheets together to form a booklet.

Step 3 **Cut Tabs**

Cut seven lines from the bottom of the top sheet, six lines from the second sheet, and so on.

Step 4 **Label**

Label each of the tabs with a lesson number. The last tab is for the vocabulary.

Reading and Writing As you read and study the chapter, use each page to write notes and to graph examples for each lesson.

What You'll Learn

- Find the slope of a line.
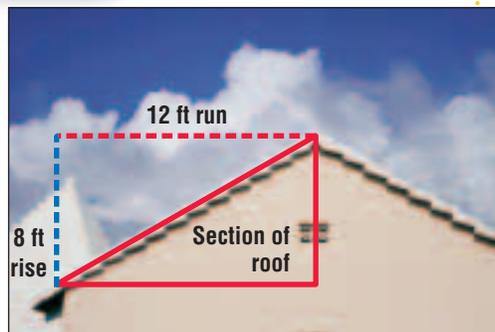- Use rate of change to solve problems.

Vocabulary

- slope
- rate of change

Why is slope important in architecture?

The slope of a roof describes how steep it is. It is the number of units the roof rises for each unit of run. In the photo, the roof rises 8 feet for each 12 feet of run.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{8}{12} \text{ or } \frac{2}{3}$$

12 ft run

8 ft rise   Section of roof

FIND SLOPE The **slope** of a line is a number determined by any two points on the line. This number describes how steep the line is. The greater the absolute value of the slope, the steeper the line. Slope is the ratio of the change in the y-coordinates (rise) to the change in the x-coordinates (run) as you move from one point to the other.

The graph shows a line that passes through (1, 3) and (4, 5).

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$= \frac{5 - 3}{4 - 1} \text{ or } \frac{2}{3}$$
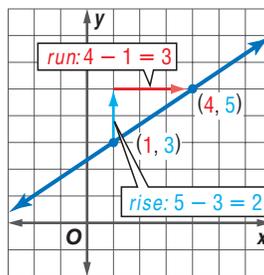
So, the slope of the line is $\frac{2}{3}$.

run: $4 - 1 = 3$

(4, 5)

(1, 3)

rise: $5 - 3 = 2$

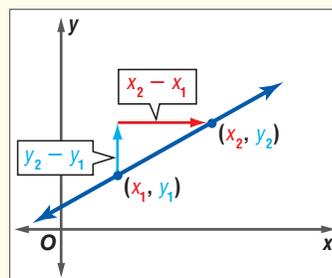**Key Concept**   **Slope of a Line**

- **Words**   The slope of a line is the ratio of the rise to the run.

- **Model**

- **Symbols**   The slope m of a nonvertical line through any two points, (x_1, y_1) and (x_2, y_2), can be found as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \leftarrow \text{change in } y$$
$$\quad\quad\quad\quad \leftarrow \text{change in } x$$

$x_2 - x_1$

$y_2 - y_1$

(x_2, y_2)

(x_1, y_1)

Study Tip

Reading Math
In x_1, the 1 is called a *subscript*. It is read *x sub 1.*

Example 1 Positive Slope

Find the slope of the line that passes through $(-1, 2)$ and $(3, 4)$.

Let $(-1, 2) = (x_1, y_1)$ and $(3, 4) = (x_2, y_2)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\dfrac{\text{rise}}{\text{run}}$

$\quad = \dfrac{4 - 2}{3 - (-1)}$ Substitute.

$\quad = \dfrac{2}{4}$ or $\dfrac{1}{2}$ Simplify.

The slope is $\dfrac{1}{2}$.

Study Tip

Common Misconception
It may make your calculations easier to choose the point on the left as (x_1, y_1). However, *either point* may be chosen as (x_1, y_1).
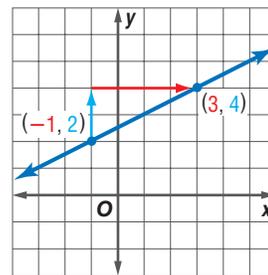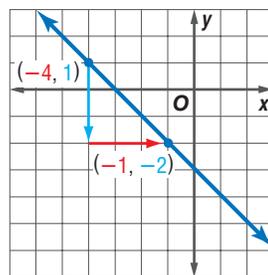
Example 2 Negative Slope

Find the slope of the line that passes through $(-1, -2)$ and $(-4, 1)$.

Let $(-1, -2) = (x_1, y_1)$ and $(-4, 1) = (x_2, y_2)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\dfrac{\text{rise}}{\text{run}}$

$\quad = \dfrac{1 - (-2)}{-4 - (-1)}$ Substitute.

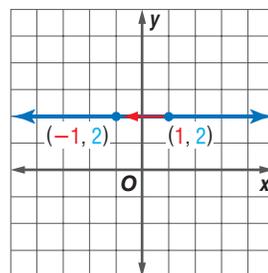$\quad = \dfrac{3}{-3}$ or -1 Simplify.

The slope is -1.

Example 3 Zero Slope

Find the slope of the line that passes through $(1, 2)$ and $(-1, 2)$.

Let $(1, 2) = (x_1, y_1)$ and $(-1, 2) = (x_2, y_2)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\dfrac{\text{rise}}{\text{run}}$

$\quad = \dfrac{2 - 2}{-1 - 1}$ Substitute.

$\quad = \dfrac{0}{-2}$ or 0 Simplify.

The slope is zero.

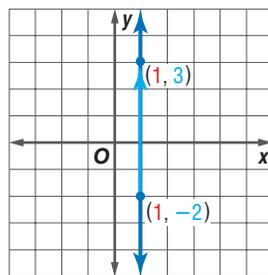**Example 4 Undefined Slope**

Find the slope of the line that passes through $(1, -2)$ and $(1, 3)$.

Let $(1, -2) = (x_1, y_1)$ and $(1, 3) = (x_2, y_2)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\dfrac{\text{rise}}{\text{run}}$

$\quad = \dfrac{3 - (-2)}{1 - 1}$ or $\dfrac{5}{0}$

Since division by zero is undefined, the slope is undefined.

www.algebra1.com/extra_examples

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Concept Summary — Classifying Lines

Positive Slope — line slopes up from left to right

Negative Slope — line slopes down from left to right

Slope of 0 — horizontal line
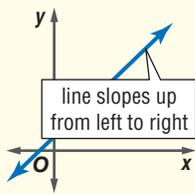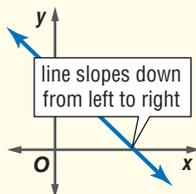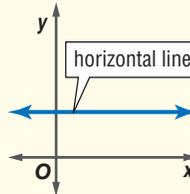
Undefined Slope — vertical line
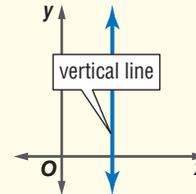
If you know the slope of a line and the coordinates of one of the points on a line, you can find the coordinates of other points on the line.

Example 5 Find Coordinates Given Slope

Find the value of r so that the line through $(r, 6)$ and $(10, -3)$ has a slope of $-\frac{3}{2}$.

Let $(r, 6) = (x_1, y_1)$ and $(10, -3) = (x_2, y_2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$-\frac{3}{2} = \frac{-3 - 6}{10 - r} \qquad \text{Substitute.}$$

$$-\frac{3}{2} = \frac{-9}{10 - r} \qquad \text{Subtract.}$$

$$-3(10 - r) = 2(-9) \qquad \text{Find the cross products.}$$

$$-30 + 3r = -18 \qquad \text{Simplify.}$$

$$-30 + 3r + 30 = -18 + 30 \qquad \text{Add 30 to each side.}$$

$$3r = 12 \qquad \text{Simplify.}$$

$$\frac{3r}{3} = \frac{12}{3} \qquad \text{Divide each side by 3.}$$

$$r = 4 \qquad \text{Simplify.}$$
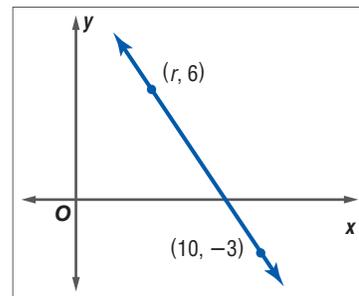
RATE OF CHANGE   Slope can be used to describe a rate of change. The **rate of change** tells, on average, how a quantity is changing over time.

Example 6 Find a Rate of Change

DINING OUT   The graph shows the amount spent on food and drink at U.S. restaurants in recent years.

a. **Find the rates of change for 1980–1990 and 1990–2000.**

   Use the formula for slope.

$$\frac{\text{rise}}{\text{run}} = \frac{\text{change in quantity}}{\text{change in time}} \quad \begin{array}{l} \leftarrow \text{billion \$} \\ \leftarrow \text{years} \end{array}$$

USA TODAY Snapshots®

Dining out
Food and drink sales at U.S. restaurants by year (in billions):

2000: $376
1990: $239
1980: $120

$300
$200
$100

'85   '90   '95

Source: National Restaurant Association

By Hilary Wasson and Alejandro Gonzalez, USA TODAY

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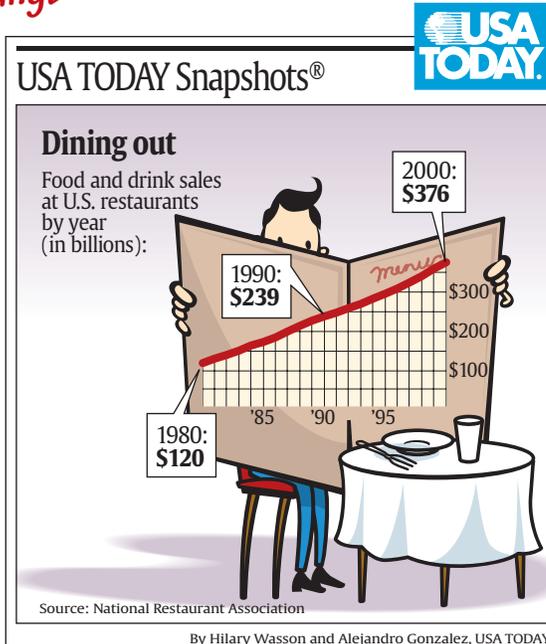
1980–1990: $\dfrac{\text{change in quantity}}{\text{change in time}} = \dfrac{239 - 120}{1990 - 1980}$ Substitute.

$$= \dfrac{119}{10} \text{ or } 11.9 \quad \text{Simplify.}$$

Spending on food and drink increased by $119 billion in a 10-year period for a rate of change of $11.9 billion per year.

1990–2000: $\dfrac{\text{change in quantity}}{\text{change in time}} = \dfrac{376 - 239}{2000 - 1990}$ Substitute.

$$= \dfrac{137}{10} \text{ or } 13.7 \quad \text{Simplify.}$$

Over this 10-year period, spending increased by $137 billion, for a rate of change of $13.7 billion per year.

b. **Explain the meaning of the slope in each case.**

For 1980–1990, on average, $11.9 billion more was spent each year than the last. For 1990–2000, on average, $13.7 billion more was spent each year than the last.

c. **How are the different rates of change shown on the graph?**

There is a greater vertical change for 1990–2000 than for 1980–1990. Therefore, the section of the graph for 1990–2000 has a steeper slope.

Check for Understanding

Concept Check

1. **Explain** how you would find the slope of the line at the right.

2. **OPEN ENDED** Draw the graph of a line having each slope.

   a. positive slope   **b.** negative slope

   c. slope of 0   **d.** undefined slope

$(-1, -3)$

$(3, -5)$

3. **Explain** why the formula for determining slope using the coordinates of two points does not apply to vertical lines.

4. **FIND THE ERROR** Carlos and Allison are finding the slope of the line that passes through $(2, 6)$ and $(5, 3)$.
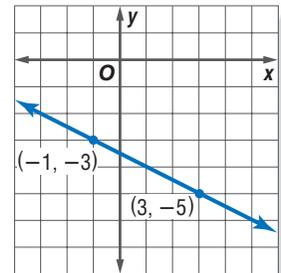
Carlos	Allison
$\dfrac{3-6}{5-2} = \dfrac{-3}{3} \text{ or } -1$	$\dfrac{6-3}{5-2} = \dfrac{3}{3} \text{ or } 1$

   Who is correct? Explain your reasoning.

Guided Practice **Find the slope of the line that passes through each pair of points.**

   5. $(1, 1), (3, 4)$ **6.** $(0, 0), (5, 4)$ **7.** $(-2, 2), (-1, -2)$

   8. $(7, -4), (9, -1)$ **9.** $(3, 5), (-2, 5)$ **10.** $(-1, 3), (-1, 0)$

   Find the value of r so the line that passes through each pair of points has the given slope.

   11. $(6, -2), (r, -6), m = 4$ **12.** $(9, r), (6, 3), m = -\dfrac{1}{3}$

◀ **CONTENTS** ▶

Application **CABLE TV** For Exercises 13 and 14, use the graph at the right.

13. Find the rate of change for 1990–1992.

14. Without calculating, find a 2-year period that had a greater rate of change than 1990–1992. Explain your reasoning.
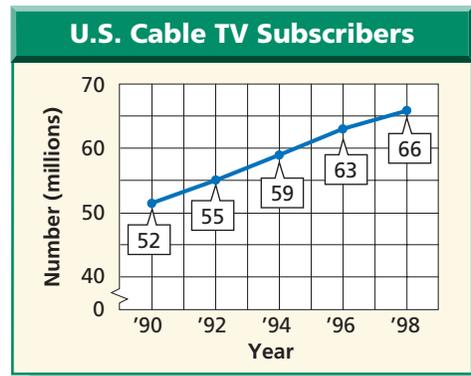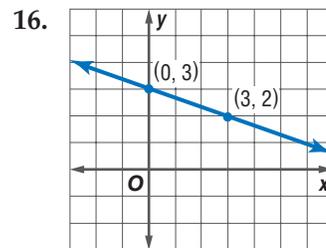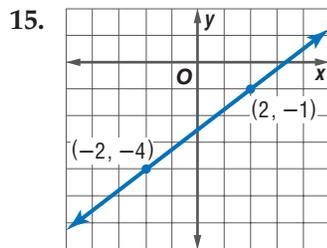
U.S. Cable TV Subscribers

Practice and Apply

Find the slope of the line that passes through each pair of points.

15.

16.

17. $(-4, -1), (-3, -3)$
18. $(-3, 3), (1, 3)$
19. $(-2, 1), (-2, 3)$
20. $(2, 3), (9, 7)$
21. $(5, 7), (-2, -3)$
22. $(-3, 6), (2, 4)$
23. $(-3, -4), (5, -1)$
24. $(2, -1), (5, -3)$
25. $(-5, 4), (-5, -1)$
26. $(2, 6), (-1, 3)$
27. $(-2, 3), (8, 3)$
28. $(-3, 9), (-7, 6)$
29. $(-8, 3), (-6, 2)$
30. $(-2, 0), (1, -1)$
31. $(4.5, -1), (5.3, 2)$
32. $(0.75, 1), (0.75, -1)$
33. $\left(2\frac{1}{2}, -1\frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}\right)$
34. $\left(\frac{3}{4}, 1\frac{1}{4}\right), \left(-\frac{1}{2}, -1\right)$

ARCHITECTURE Use a ruler to estimate the slope of each roof.

35.

36.
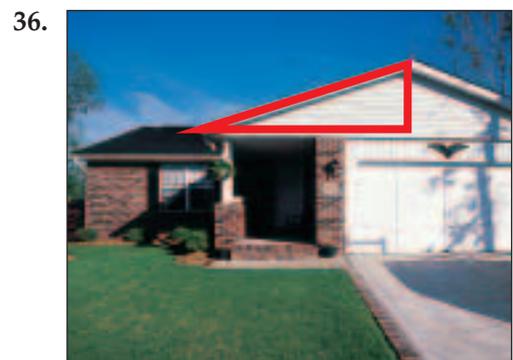
37. Find the slope of the line that passes through the origin and (r, s).

38. What is the slope of the line that passes through (a, b) and $(a, -b)$?

39. **PAINTING** A ladder reaches a height of 16 feet on a wall. If the bottom of the ladder is placed 4 feet away from the wall, what is the slope of the ladder as a positive number?

◀ **CONTENTS** ▶

40. PART-TIME JOBS In 1991, the federal minimum wage rate was $4.25 per hour. In 1997, it was increased to $5.15. Find the annual rate of change in the federal minimum wage rate from 1991 to 1997.

Find the value of *r* so the line that passes through each pair of points has the given slope.

41. $(6, 2), (9, r), m = -1$

42. $(4, -5), (3, r), m = 8$

43. $(5, r), (2, -3), m = \dfrac{4}{3}$

44. $(-2, 7), (r, 3), m = \dfrac{4}{3}$

45. $\left(\dfrac{1}{2}, -\dfrac{1}{4}\right), \left(r, -\dfrac{5}{4}\right), m = 4$

46. $\left(\dfrac{2}{3}, r\right), \left(1, \dfrac{1}{2}\right), m = \dfrac{1}{2}$

47. $(4, r), (r, 2), m = -\dfrac{5}{3}$

48. $(r, 5), (-2, r), m = -\dfrac{2}{9}$

49. CRITICAL THINKING Explain how you know that the slope of the line through $(-4, -5)$ and $(4, 5)$ is positive without calculating.

HEALTH For Exercises 50–52, use the table that shows Karen's height from age 12 to age 20.

Age (years)	12	14	16	18	20
Height (inches)	60	64	66	67	67

50. Make a broken-line graph of the data.

51. Use the graph to determine the two-year period when Karen grew the fastest. Explain your reasoning.

52. Explain the meaning of the horizontal section of the graph.

SCHOOL For Exercises 53–55, use the graph that shows public school enrollment.

53. For which 5-year period was the rate of change the greatest? When was the rate of change the least?

54. Find the rate of change from 1985 to 1990.

55. Explain the meaning of the part of the graph with a negative slope.

U.S. Public School Enrollment Grades 9–12
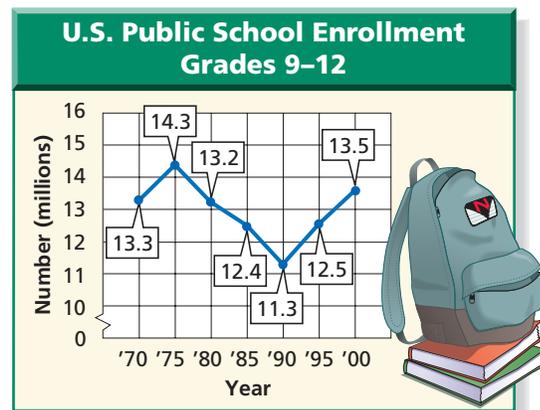
56. RESEARCH Use the Internet or other reference to find the population of your city or town in 1930, 1940, . . . , 2000. For which decade was the rate of change the greatest?

57. CONSTRUCTION The slope of a stairway determines how easy it is to climb the stairs. Suppose the vertical distance between two floors is 8 feet 9 inches. Find the total run of the ideal stairway in feet and inches.

58.   Answer the question that was posed at the beginning of the lesson.

Why is slope important in architecture?

Include the following in your answer:
- an explanation of how to find the slope of a roof, and
- a comparison of the appearance of roofs with different slopes.

59. The slope of the line passing through $(5, -4)$ and $(5, -10)$ is
Ⓐ positive.   Ⓑ negative.   Ⓒ zero.   Ⓓ undefined.

60. The slope of the line passing through (a, b) and (c, d) is
Ⓐ $\dfrac{d-c}{b-a}$.   Ⓑ $\dfrac{b-d}{a-c}$.   Ⓒ $\dfrac{d-b}{a-c}$.   Ⓓ $\dfrac{a-c}{b-d}$.

Extending the Lesson

61. Choose four different pairs of points from those labeled on the graph. Find the slope of the line using the coordinates of each pair of points. Describe your findings.

62. **MAKE A CONJECTURE**   Determine whether $Q(2, 3)$, $R(-1, -1)$, and $S(-4, -2)$ lie on the same line. Explain your reasoning.

Maintain Your Skills

Mixed Review   **Write an equation for each function.**   *(Lesson 4-8)*

63.

x	1	2	3	4	5
f(x)	5	10	15	20	25

64.

x	-2	-1	1	2	4
f(x)	13	12	10	9	7

Determine whether each relation is a function.   *(Lesson 4-6)*

65. $y = -15$

66. $x = 5$

67. $\{(1, 0), (1, 4), (-1, 1)\}$

68. $\{(6, 3), (5, -2), (2, 3)\}$

69. Graph $x - y = 0$.   *(Lesson 4-4)*

70. What number is 40% of 37.5?   *(Prerequisite Skill)*

Find each product.   *(Lesson 2-4)*

71. $7(-3)$

72. $(-4)(-2)$

73. $(9)(-4)$

74. $(-8)(3.7)$

75. $\left(-\dfrac{7}{8}\right)\left(\dfrac{1}{3}\right)$

76. $\left(\dfrac{1}{4}\right)\left(\dfrac{1}{2}\right)(-14)$

Getting Ready for the Next Lesson   **PREREQUISITE SKILL   Find each quotient.**
*(To review **dividing fractions**, see pages 800 and 801.)*

77. $6 \div \dfrac{2}{3}$

78. $12 \div \dfrac{1}{4}$

79. $10 \div \dfrac{3}{8}$

80. $\dfrac{1}{2} \div \dfrac{1}{3}$

81. $\dfrac{3}{4} \div \dfrac{1}{6}$

82. $\dfrac{3}{4} \div 6$

83. $18 \div \dfrac{7}{8}$

84. $\dfrac{3}{8} \div \dfrac{2}{5}$

85. $2\dfrac{2}{3} \div \dfrac{1}{4}$

Reading Mathematics

Mathematical Words and Everyday Words

You may have noticed that many words used in mathematics are also used in everyday language. You can use the everyday meaning of these words to better understand their mathematical meaning. The table shows two mathematical words along with their everyday and mathematical meanings.

Word	Everyday Meaning	Mathematical Meaning
expression	1. something that expresses or communicates in words, art, music, or movement 2. the manner in which one expresses oneself, especially in speaking, depicting, or performing	one or more numbers or variables along with one or more arithmetic operations
function	1. the action for which one is particularly fitted or employed 2. an official ceremony or a formal social occasion 3. something closely related to another thing and dependent on it for its existence, value, or significance	a relationship in which the output depends upon the input

Source: *The American Heritage Dictionary of the English Language*

Notice that the mathematical meaning is more specific, but related to the everyday meaning. For example, the mathematical meaning of *expression* is closely related to the first everyday definition. In mathematics, an expression communicates using symbols.

Reading to Learn

1. How does the mathematical meaning of *function* compare to the everyday meaning?

2. **RESEARCH** Use the Internet or other reference to find the everyday meaning of each word below. How might these words apply to mathematics? Make a table like the one above and note the mathematical meanings that you learn as you study Chapter 5.
   a. slope
   b. intercept
   c. parallel

5-2 Slope and Direct Variation

- Write and graph direct variation equations.
- Solve problems involving direct variation.

Vocabulary
- direct variation
- constant of variation
- family of graphs
- parent graph

How is slope related to your shower?

A standard showerhead uses about 6 gallons of water per minute. If you graph the ordered pairs from the table, the slope of the line is 6.

x (minutes)	y (gallons)
0	0
1	6
2	12
3	18
4	24

Gallons of Water Used in a Shower

The equation is $y = 6x$.
The number of gallons of water y depends *directly* on the amount of time in the shower x.
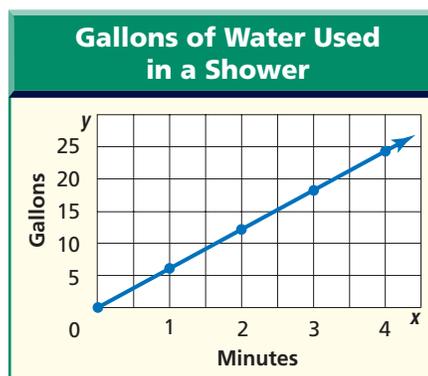
DIRECT VARIATION A **direct variation** is described by an equation of the form $y = kx$, where $k \neq 0$. We say that *y varies directly with x* or *y varies directly as x*. In the equation $y = kx$, k is the **constant of variation**.

Example 1 Slope and Constant of Variation

Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.

a.
$(1, 3)$
$(0, 0)$
$y = 3x$
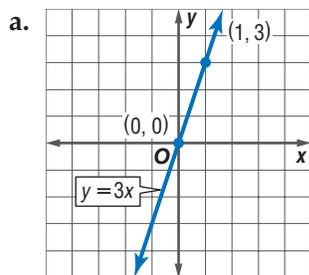
The constant of variation is 3.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \dfrac{3 - 0}{1 - 0}$ $(x_1, y_1) = (0, 0)$
$(x_2, y_2) = (1, 3)$

$m = 3$ The slope is 3.

b.
$y = -2x$
$(0, 0)$
$(1, -2)$
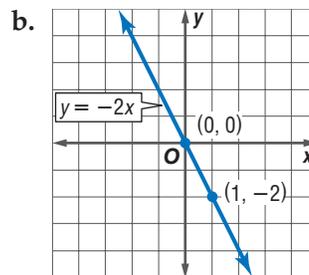
The constant of variation is -2.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \dfrac{-2 - 0}{1 - 0}$ $(x_1, y_1) = (0, 0)$
$(x_2, y_2) = (1, -2)$

$m = -2$ The slope is -2.

Compare the constant of variation with the slope of the graph for each example. Notice that the slope of the graph of $y = kx$ is k.

CONTENTS

The ordered pair (0, 0) is a solution of $y = kx$. Therefore, the graph of $y = kx$ passes through the origin. You can use this information to graph direct variation equations.

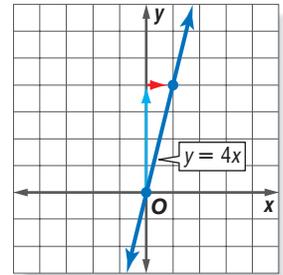**Example 2 Direct Variation with K > 0**
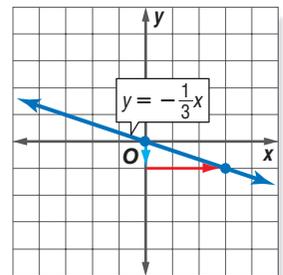
Graph $y = 4x$.

Step 1 Write the slope as a ratio.

$$4 = \frac{4}{1} \quad \frac{\text{rise}}{\text{run}}$$

Step 2 Graph (0, 0).

Step 3 From the point (0, 0), move up 4 units and right 1 unit. Draw a dot.

Step 4 Draw a line containing the points.

Example 3 Direct Variation with K < 0

Graph $y = -\frac{1}{3}x$.

Step 1 Write the slope as a ratio.

$$-\frac{1}{3} = \frac{-1}{3} \quad \frac{\text{rise}}{\text{run}}$$

Step 2 Graph (0, 0).

Step 3 From the point (0, 0), move down 1 unit and right 3 units. Draw a dot.

Step 4 Draw a line containing the points.

A **family of graphs** includes graphs and equations of graphs that have at least one characteristic in common. The **parent graph** is the simplest graph in a family.
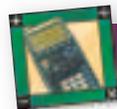
Graphing Calculator Investigation

Family of Graphs

The calculator screen shows the graphs of $y = x$, $y = 2x$, and $y = 4x$.

Think and Discuss

1. Describe any similarities among the graphs.

2. Describe any differences among the graphs.

3. Write an equation whose graph has a steeper slope than $y = 4x$. Check your answer by graphing $y = 4x$ and your equation.

4. Write an equation whose graph lies between the graphs of $y = x$ and $y = 2x$. Check your answer by graphing the equations.

5. Write a description of this family of graphs. What characteristics do the graphs have in common? How are they different?

6. The equations whose graphs are in this family are all of the form $y = mx$. How does the graph change as the absolute value of m increases?
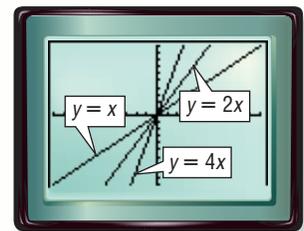
[−10, 10] scl: 1 by [−10, 10] scl: 1

- Direct variation equations are of the form $y = kx$, where $k \neq 0$.
- The graph of $y = kx$ always passes through the origin.

• The slope can be positive. $k > 0$	• The slope can be negative. $k < 0$
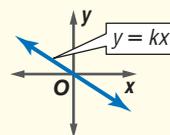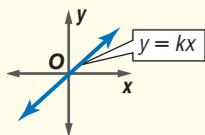
If you know that y varies directly as x, you can write a direct variation equation that relates the two quantities.

Example 4 — Write and Solve a Direct Variation Equation

Suppose y varies directly as x, and $y = 28$ when $x = 7$.

a. Write a direct variation equation that relates x and y.

Find the value of k.

$y = kx$ Direct variation formula

$28 = k(7)$ Replace y with 28 and x with 7.

$\dfrac{28}{7} = \dfrac{k(7)}{7}$ Divide each side by 7.

$4 = k$ Simplify.

Therefore, $y = 4x$.

b. Use the direct variation equation to find x when $y = 52$.

$y = 4x$ Direct variation equation

$52 = 4x$ Replace y with 52.

$\dfrac{52}{4} = \dfrac{4x}{4}$ Divide each side by 4.

$13 = x$ Simplify.

Therefore, $x = 13$ when $y = 52$.

SOLVE PROBLEMS   One of the most common uses of direct variation is the formula for distance, $d = rt$. In the formula, distance d varies directly as time t, and the rate r is the constant of variation.

Example 5 — Direct Variation Equation

BIOLOGY   **A flock of snow geese migrated 375 miles in 7.5 hours.**

a. Write a direct variation equation for the distance flown in any time.

Words   The distance traveled is 375 miles, and the time is 7.5 hours.

Variables   Let r = rate.

	Distance	equals	rate	times	time.
Equation	375 mi	=	r	×	7.5 h

Solve for the rate.

$375 = r(7.5)$ Original equation

$\dfrac{375}{7.5} = \dfrac{r(7.5)}{7.5}$ Divide each side by 7.5.

$50 = r$ Simplify.

Therefore, the direct variation equation is $d = 50t$.

More About...

Biology

Snow geese migrate more than 3000 miles from their winter home in the southwest United States to their summer home in the Canadian arctic.

Source: Audubon Society

CONTENTS

b. Graph the equation.

The graph of $d = 50t$ passes through the origin with slope 50.

$$m = \frac{50}{1} \quad \begin{matrix} \text{rise} \\ \text{run} \end{matrix}$$

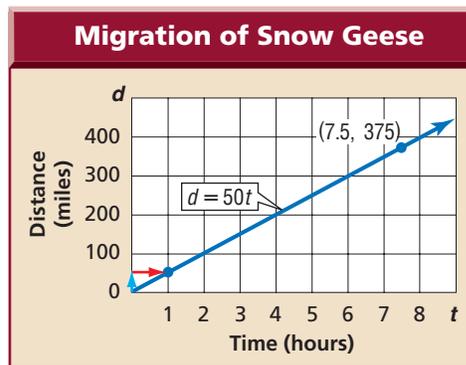**c. Estimate how many hours of flying time it would take the geese to migrate 3000 miles.**

$$\begin{aligned} d &= 50t && \text{Original equation} \\ 3000 &= 50t && \text{Replace } d \text{ with 3000.} \\ \frac{3000}{50} &= \frac{50t}{50} && \text{Divide each side by 50.} \\ t &= 60 && \text{Simplify.} \end{aligned}$$

At this rate, it will take 60 hours of flying time to migrate 3000 miles.

Migration of Snow Geese

Check for Understanding

Concept Check
1. **OPEN ENDED** Write a general equation for *y varies directly as x.*

2. **Choose** the equations that represent direct variations. Then find the constant of variation for each direct variation.

   a. $15 = rs$ **b.** $4a = b$ **c.** $z = \frac{1}{3}x$ **d.** $s = \frac{9}{t}$

3. **Explain** how the constant of variation and the slope are related in a direct variation equation.

Guided Practice
Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

4.

5.
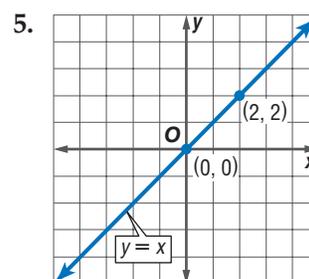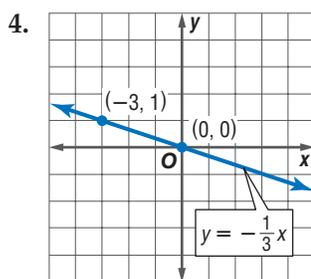
Graph each equation.

6. $y = 2x$ 7. $y = -3x$ 8. $y = \frac{1}{2}x$

Write a direct variation equation that relates x and y. Assume that y varies directly as x. Then solve.

9. If $y = 27$ when $x = 6$, find x when $y = 45$.

10. If $y = 10$ when $x = 9$, find x when $y = 9$.

11. If $y = -7$ when $x = -14$, find y when $x = 20$.

Application
JOBS For Exercises 12–14, use the following information.
Suppose you work at a job where your pay varies directly as the number of hours you work. Your pay for 7.5 hours is $45.

12. Write a direct variation equation relating your pay to the hours worked.

13. Graph the equation.
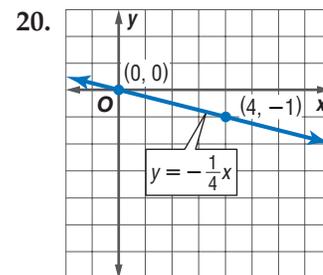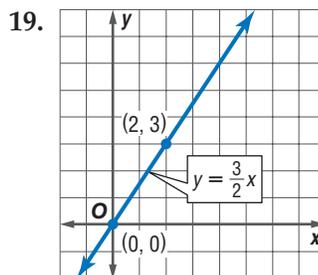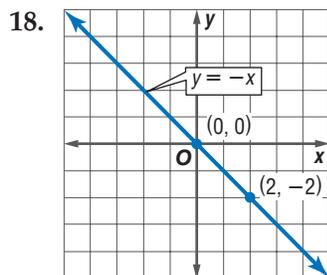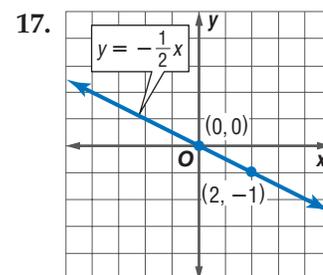
14. Find your pay if you work 30 hours.

CONTENTS

Practice and Apply

Name the constant of variation for each equation. Then determine the slope of the line that passes through each pair of points.

15.

$y = 2x$, $(2, 4)$, O, $(0, 0)$

16.

O $(0, 0)$, $(-1, -4)$, $y = 4x$

17.

$y = -\frac{1}{2}x$, $(0, 0)$, O, $(2, -1)$

18.

$y = -x$, $(0, 0)$, O, $(2, -2)$

19.

$(2, 3)$, $y = \frac{3}{2}x$, O, $(0, 0)$

20.

$(0, 0)$, O, $(4, -1)$ x, $y = -\frac{1}{4}x$
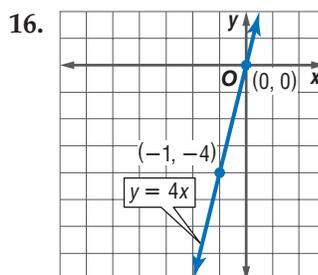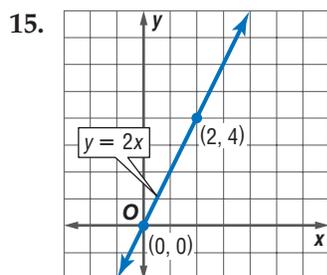
Graph each equation.

21. $y = x$

22. $y = 3x$

23. $y = -x$

24. $y = -4x$

25. $y = \frac{1}{4}x$

26. $y = \frac{3}{5}x$

27. $y = \frac{5}{2}x$

28. $y = \frac{7}{5}x$

29. $y = \frac{1}{5}x$

30. $y = -\frac{2}{3}x$

31. $y = -\frac{4}{3}x$

32. $y = -\frac{9}{2}x$

Write a direct variation equation that relates x and y. Assume that y varies directly as x. Then solve.

33. If $y = 8$ when $x = 4$, find y when $x = 5$.

34. If $y = 36$ when $x = 6$, find x when $y = 42$.

35. If $y = -16$ when $x = 4$, find x when $y = 20$.

36. If $y = -18$ when $x = 6$, find x when $y = 6$.

37. If $y = 4$ when $x = 12$, find y when $x = -24$.

38. If $y = 12$ when $x = 15$, find x when $y = 21$.

39. If $y = 2.5$ when $x = 0.5$, find y when $x = 20$.

40. If $y = -6.6$ when $x = 9.9$, find y when $x = 6.6$.

41. If $y = 2\frac{2}{3}$ when $x = \frac{1}{4}$, find y when $x = 1\frac{1}{8}$.

42. If $y = 6$ when $x = \frac{2}{3}$, find x when $y = 12$.

Write a direct variation equation that relates the variables. Then graph the equation.

43. **GEOMETRY** The circumference C of a circle is about 3.14 times the diameter d.

44. **GEOMETRY** The perimeter P of a square is 4 times the length of a side s.

45. **SEWING** The total cost is C for n yards of ribbon priced at $0.99 per yard.

46. **RETAIL** Kona coffee beans are $14.49 per pound. The total cost of p pounds is C.

CONTENTS

47. CRITICAL THINKING Suppose y varies directly as x. If the value of x is doubled, what happens to the value of y? Explain.

BIOLOGY Which line in the graph represents the sprinting speeds of each animal?

48. elephant, 25 mph

49. reindeer, 32 mph

50. lion, 50 mph

51. grizzly bear, 30 mph

Sprinting Speeds
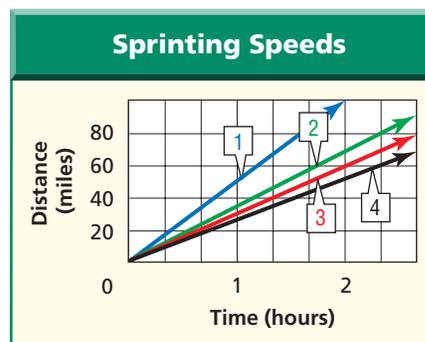
SPACE For Exercises 52 and 53, use the following information.
The weight of an object on the moon varies directly with its weight on Earth. With all of his equipment, astronaut Neil Armstrong weighed 360 pounds on Earth, but weighed only 60 pounds on the moon.

52. Write an equation that relates weight on the moon m with weight on Earth e.

53. Suppose you weigh 138 pounds on Earth. What would you weigh on the moon?

ANIMALS For Exercises 54 and 55, use the following information.
Most animals age more rapidly than humans do. The chart shows equivalent ages for horses and humans.

Horse age (x)	0	1	2	3	4	5
Human age (y)	0	3	6	9	12	15

54. Write an equation that relates human age to horse age.

55. Find the equivalent horse age for a human who is 16 years old.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is slope related to your shower?

Include the following in your answer:

- an equation that relates the number of gallons y to the time spent in the shower x for a low-flow showerhead that uses only 2.5 gallons of water per minute, and

- a comparison of the steepness of the graph of this equation to the graph at the top of page 264.

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

57. Which equation best describes the graph at the right?

Ⓐ $y = 2x$ Ⓑ $y = -2x$

Ⓒ $y = \frac{1}{2}x$ Ⓓ $y = -\frac{1}{2}x$
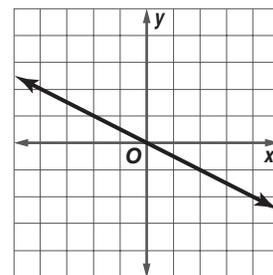
58. Which equation does *not* model a direct variation?

Ⓐ $y = 4x$ Ⓑ $y = 22x$

Ⓒ $y = 3x + 1$ Ⓓ $y = \frac{1}{2}x$

Graphing Calculator

FAMILIES OF GRAPHS For Exercises 59–62, use the graphs of $y = -1x$, $y = -2x$, and $y = -4x$, which form a family of graphs.

59. Graph $y = -1x$, $y = -2x$, and $y = -4x$ on the same screen.

60. How are these graphs similar to the graphs in the Graphing Calculator Investigation on page 265? How are they different?

www.algebra1.com/self_check_quiz

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61. Write an equation whose graph has a steeper slope than $y = -4x$.

62. MAKE A CONJECTURE Explain how you can tell without graphing which of two direct variation equations has the graph with a steeper slope.

Maintain Your Skills

Mixed Review **Find the slope of the line that passes through each pair of points.** *(Lesson 5-1)*

63.

(1, 3)
(2, 0)

64.

(2, 2)
(2, −2)

65.

(−2, 3)
(−3, 1)
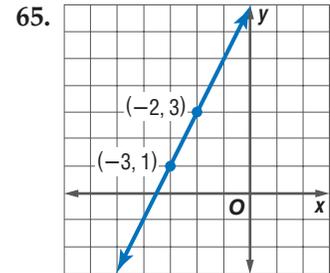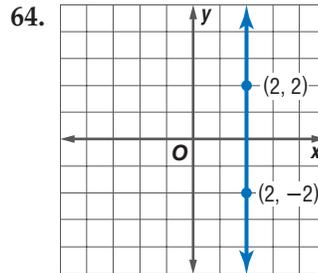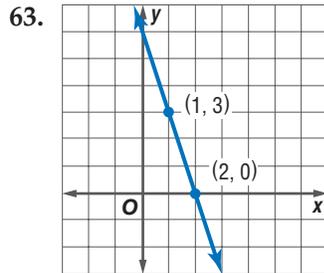
66. Find the value of r so that the line that passes through (1, 7) and (r, 3) has a slope of 2. *(Lesson 5-1)*

Each table below represents points on a linear graph. Copy and complete each table.
(Lesson 4-8)

67.

x	0	1	2	3	4	5
y	1		9	13		21

68.

x	2	4	6	8	10	12
y			4	2		-2

Add or subtract. *(Lesson 2-2)*

69. $15 + (-12)$ **70.** $8 - (-5)$ **71.** $-9 - 6$ **72.** $-18 - 12$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Solve each equation for y.
*(To review **rewriting equations**, see Lesson 3-8.)*

73. $-3x + y = 8$ **74.** $2x + y = 7$ **75.** $4x = y + 3$

76. $2y = 4x + 10$ **77.** $9x + 3y = 12$ **78.** $x - 2y = 5$

Practice Quiz 1   Lessons 5-1 and 5-2

Find the slope of the line that passes through each pair of points. *(Lesson 5-1)*

1. $(-4, -6), (-3, -8)$ **2.** $(8, 3), (-11, 3)$ **3.** $(-4, 8), (5, 9)$ **4.** $(0, 1), (7, 11)$

Find the value of r so the line that passes through each pair of points has the given slope. *(Lesson 5-1)*

5. $(5, -3), (r, -5), m = 2$ **6.** $(6, r), (-4, 9), m = \dfrac{3}{2}$

Graph each equation. *(Lesson 5-2)*

7. $y = -7x$ **8.** $y = \dfrac{3}{4}x$

Write a direct variation equation that relates x and y. Assume that y varies directly as x. Then solve. *(Lesson 5-2)*

9. If $y = 24$ when $x = 8$, find y when $x = -3$. **10.** If $y = -10$ when $x = 15$, find x when $y = -6$.

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Algebra Activity

Algebra Activity

A Preview of Lesson 5-3

Investigating Slope-Intercept Form

Collect the Data
- Cut a small hole in a top corner of a plastic sandwich bag. Loop a long rubber band through the hole.
- Tape the free end of the rubber band to the desktop.
- Use a centimeter ruler to measure the distance from the desktop to the end of the bag. Record this distance for 0 washers in the bag using a table like the one below.

Number of Washers x	Distance y
0	
1	
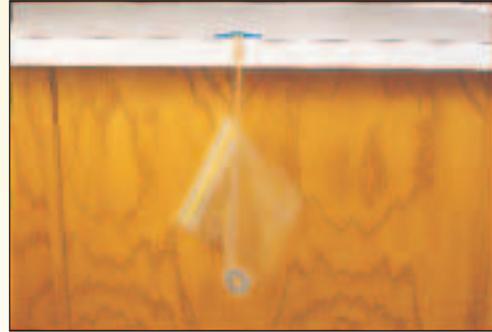
- Place one washer in the plastic bag. Then measure and record the new distance from the desktop to the end of the bag.
- Repeat the experiment, adding different numbers of washers to the bag. Each time, record the number of washers and the distance from the desktop to the end of the bag.

Analyze the Data
1. The domain contains values represented by the independent variable, washers. The range contains values represented by the dependent variable, distance. On grid paper, graph the ordered pairs (washers, distance).
2. Write a sentence that describes the points on the graph.
3. Describe the point that represents the trial with no washers in the bag.
4. The rate of change can be found by using the formula for slope.
   $$\frac{\text{rise}}{\text{run}} = \frac{\text{change in distance}}{\text{change in number of washers}}$$
   Find the rate of change in the distance from the desktop to the end of the bag as more washers are added.
5. Explain how the rate of change is shown on the graph.

Make a Conjecture
The graph shows sample data from a rubber band experiment. Draw a graph for each situation.
6. A bag that hangs 10.5 centimeters from the desktop when empty and lengthens at the rate of the sample.
7. A bag that has the same length when empty as the sample and lengthens at a faster rate.
8. A bag that has the same length when empty as the sample and lengthens at a slower rate.

Algebra Activity   Investigating Slope-Intercept Form   271

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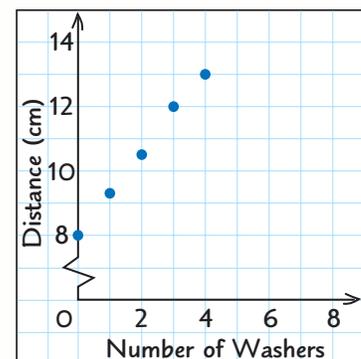
5-3 Slope-Intercept Form

What You'll Learn

- Write and graph linear equations in slope-intercept form.
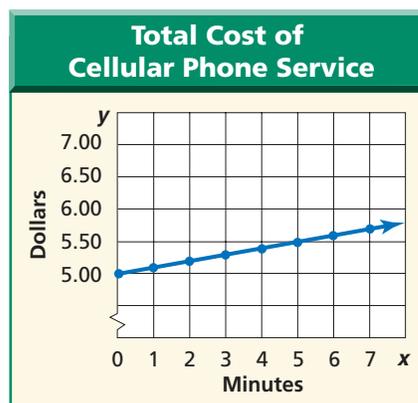- Model real-world data with an equation in slope-intercept form.

Vocabulary
- slope-intercept form

How is a *y*-intercept related to a flat fee?

A cellular phone service provider charges $0.10 per minute plus a flat fee of $5.00 each month.

x (minutes)	y (dollars)
0	5.00
1	5.10
2	5.20
3	5.30
4	5.40
5	5.50
6	5.60
7	5.70

Total Cost of Cellular Phone Service

The slope of the line is 0.1. It crosses the *y*-axis at (0, 5).
The equation of the line is $y = 0.1x + 5$.

charge per minute, $0.10 ⬐   ⬐ flat fee, $5.00

SLOPE-INTERCEPT FORM   An equation of the form $y = mx + b$ is in **slope-intercept form**. When an equation is written in this form, you can identify the slope and *y*-intercept of its graph.
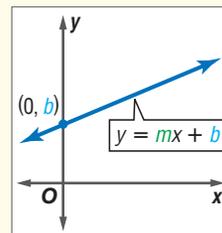
Key Concept   Slope-Intercept Form

- **Words**   The linear equation $y = mx + b$ is written in slope-intercept form, where *m* is the slope and *b* is the *y*-intercept.

- **Symbols** $y = mx + b$
    slope ⬐   ⬐ y-intercept

- **Model**

    (0, b)

    $y = mx + b$

Example 1   Write an Equation Given Slope and y-Intercept

Write an equation of the line whose slope is 3 and whose *y*-intercept is 5.
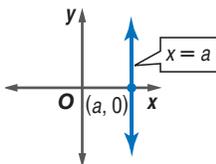
$y = mx + b$ Slope-intercept form

$y = 3x + 5$ Replace *m* with 3 and *b* with 5.

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Example 2 — Write an Equation Given Two Points

Write an equation of the line shown in the graph.

Step 1 You know the coordinates of two points on the line. Find the slope. Let $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (2, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{\text{rise}}{\text{run}}$$

$$m = \frac{-1 - 3}{2 - 0} \qquad \begin{array}{l} x_1 = 0, x_2 = 2 \\ y_1 = 3, y_2 = -1 \end{array}$$

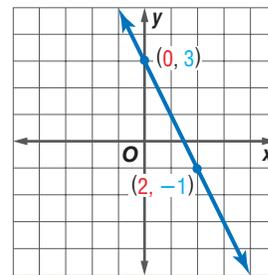$$m = \frac{-4}{2} \text{ or } -2 \quad \text{Simplify.}$$

The slope is -2.

Step 2 The line crosses the y-axis at $(0, 3)$. So, the y-intercept is 3.

Step 3 Finally, write the equation.

$$y = mx + b \qquad \text{Slope-intercept form}$$

$$y = -2x + 3 \qquad \text{Replace } m \text{ with } -2 \text{ and } b \text{ with 3.}$$

The equation of the line is $y = -2x + 3$.

One advantage of the slope-intercept form is that it allows you to graph an equation quickly.

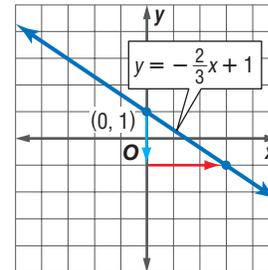## Example 3 — Graph an Equation in Slope-Intercept Form

Graph $y = -\frac{2}{3}x + 1$.

Step 1 The y-intercept is 1. So, graph $(0, 1)$.

Step 2 The slope is $-\frac{2}{3}$ or $\frac{-2}{3}$. $\quad \frac{\text{rise}}{\text{run}}$

From $(0, 1)$, move down 2 units and right 3 units. Draw a dot.

Step 3 Draw a line connecting the points.

Example 4 — Graph an Equation in Standard Form

Graph $5x - 3y = 6$.

Step 1 Solve for y to find the slope-intercept form.

$5x - 3y = 6$	Original equation
$5x - 3y - 5x = 6 - 5x$	Subtract $5x$ from each side.
$-3y = 6 - 5x$	Simplify.
$-3y = -5x + 6$	$6 - 5x = 6 + (-5x)$ or $-5x + 6$
$\dfrac{-3y}{-3} = \dfrac{-5x + 6}{-3}$	Divide each side by -3.
$\dfrac{-3y}{-3} = \dfrac{-5x}{-3} + \dfrac{6}{-3}$	Divide each term in the numerator by -3.
$y = \dfrac{5}{3}x - 2$	Simplify.

(continued on the next page)

Study Tip

Vertical Lines
The equation of a vertical line *cannot* be written in slope-intercept form. *Why?*

$x = a$

$O \ (a, 0) \ x$

Horizontal Lines
The equation of a horizontal line *can* be written in slope-intercept form as $y = 0x + b$ or $y = b$.

$y = b$

$(0, b)$

www.algebra1.com/extra_examples
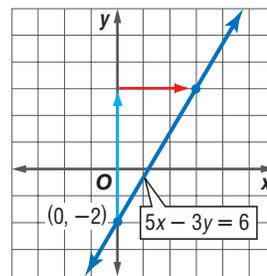
CONTENTS

Step 2 The y-intercept of $y = \frac{5}{3}x - 2$ is -2.
So, graph $(0, -2)$.

Step 3 The slope is $\frac{5}{3}$. From $(0, -2)$, move up 5 units
and right 3 units. Draw a dot.

Step 4 Draw a line containing the points.

(0, −2) $5x - 3y = 6$

MODEL REAL-WORLD DATA If a quantity changes at a constant rate over time, it can be modeled by a linear equation. The y-intercept represents a starting point, and the slope represents the rate of change.

Example 5 Write an Equation in Slope-Intercept Form

AGRICULTURE The natural sweeteners used in foods include sugar, corn sweeteners, syrup, and honey. Use the information at the left about natural sweeteners.

a. The amount of natural sweeteners consumed has increased by an average of 2.6 pounds per year. Write a linear equation to find the average consumption of natural sweeteners in any year after 1989.

Words   The consumption increased 2.6 pounds per year, so the rate of change is 2.6 pounds per year. In the first year, the average consumption was 133 pounds.

Variables Let C = average consumption.
Let n = number of years after 1989.

Equation

Average consumption	equals	rate of change	times	number of years after 1989	plus	amount at start.
C	$=$	2.6	$\cdot$	n	$+$	133

b. Graph the equation.

The graph passes through $(0, 133)$ with slope 2.6.

c. Find the number of pounds of natural sweeteners consumed by each person in 1999.

The year 1999 is 10 years after 1989. So, $n = 10$.

$C = 2.6n + 133$ Consumption equation

$C = 2.6(10) + 133$ Replace n with 10.

$C = 159$ Simplify.

Consumption of Natural Sweeteners

(10, 159)

$C = 2.6n + 133$

Pounds

Years Since 1989

So, the average person consumed 159 pounds of natural sweeteners in 1999.

CHECK Notice that $(10, 159)$ lies on the graph.

CONTENTS
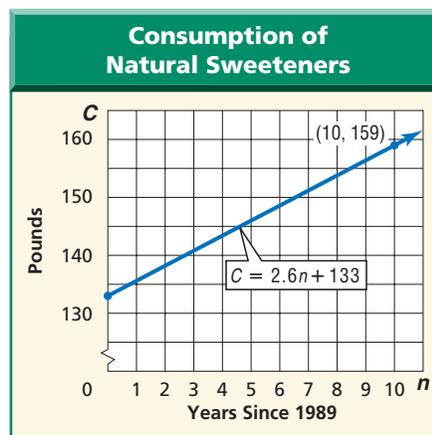
## Check for Understanding

Concept Check   1. **OPEN ENDED**   Write an equation for a line with a slope of 7.

2. **Explain** why equations of vertical lines cannot be written in slope-intercept form, but equations of horizontal lines can.

3. **Tell** which part of the slope-intercept form represents the rate of change.

Guided Practice   **Write an equation of the line with the given slope and y-intercept.**

4. slope: -3, y-intercept: 1

5. slope: 4, y-intercept: -2

Write an equation of the line shown in each graph.

6.

7.

Graph each equation.

8. $y = 2x - 3$

9. $y = -3x + 1$
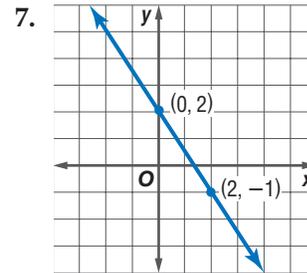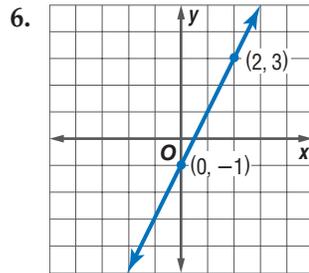
10. $2x + y = 5$

Application   **MONEY   For Exercises 11–13, use the following information.**
Suppose you have already saved $50 toward the cost of a new television set. You plan to save $5 more each week for the next several weeks.

11. Write an equation for the total amount T you will have w weeks from now.

12. Graph the equation.

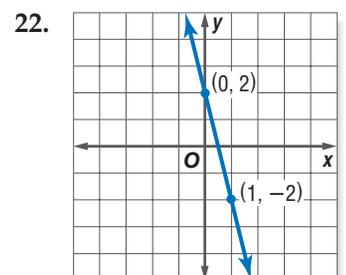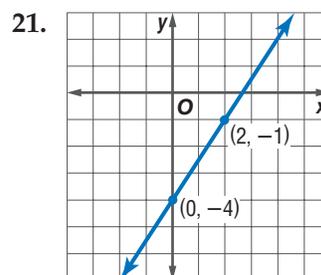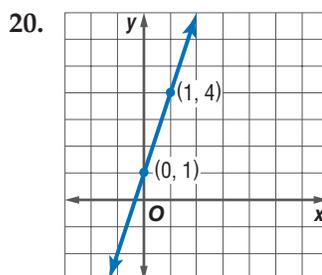13. Find the total amount saved after 7 weeks.

Practice and Apply

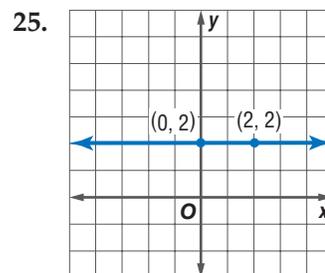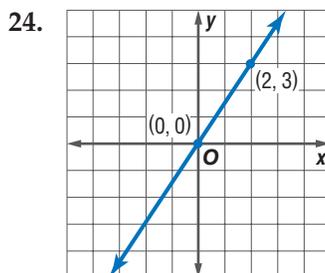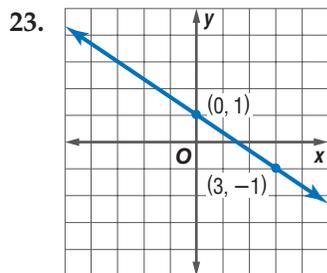**Write an equation of the line with the given slope and y-intercept.**

14. slope: 2, y-intercept: -6

15. slope: 3, y-intercept: -5

16. slope: $\frac{1}{2}$, y-intercept: 3

17. slope: $-\frac{3}{5}$, y-intercept: 0

18. slope: -1, y-intercept: 10

19. slope: 0.5; y-intercept: 7.5

Write an equation of the line shown in each graph.

20.

21.

22.

Write an equation of the line shown in each graph.

23.

24.

25.

26. Write an equation of a horizontal line that crosses the y-axis at $(0, -5)$.

27. Write an equation of a line that passes through the origin with slope 3.

Graph each equation.

28. $y = 3x + 1$
29. $y = x - 2$
30. $y = -4x + 1$

31. $y = -x + 2$
32. $y = \frac{1}{2}x + 4$
33. $y = -\frac{1}{3}x - 3$

34. $3x + y = -2$
35. $2x - y = -3$
36. $3y = 2x + 3$

37. $-2y = 6x - 4$
38. $2x + 3y = 6$
39. $4x - 3y = 3$

Write a linear equation in slope-intercept form to model each situation.

40. You rent a bicycle for $20 plus $2 per hour.

41. An auto repair shop charges $50 plus $25 per hour.

42. A candle is 6 inches tall and burns at a rate of $\frac{1}{2}$ inch per hour.

**43.** The temperature is 15° and is expected to fall 2° each hour during the night.

44. CRITICAL THINKING The equations $y = 2x + 3$, $y = 4x + 3$, $y = -x + 3$, and $y = -10x + 3$ form a family of graphs. What characteristic do their graphs have in common?

SALES For Exercises 45 and 46, use the following information and the graph at the right.
In 1991, book sales in the United States totaled $16 billion. Sales increased by about $1 billion each year until 1999.

45. Write an equation to find the total sales S for any year t between 1991 and 1999.

46. If the trend continues, what will sales be in 2005?

Book Sales
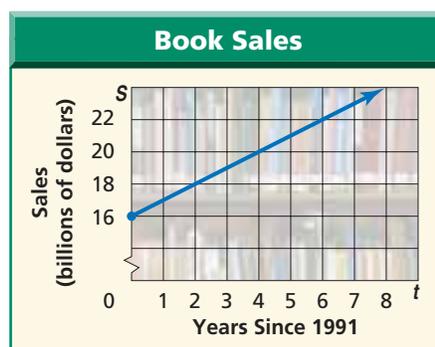
Source: Association of American Publishers

TRAFFIC For Exercises 47–49, use the following information.
In 1966, the traffic fatality rate in the United States was 5.5 fatalities per 100 million vehicle miles traveled. Between 1966 and 1999, the rate decreased by about 0.12 each year.

47. Write an equation to find the fatality rate R for any year t between 1966 and 1999.

48. Graph the equation.

49. Find the fatality rate in 1999.

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50. WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

How is a *y*-intercept related to a flat fee?

Include the following in your answer:

- the point at which the graph would cross the *y*-axis if your cellular phone service provider charges a rate of $0.07 per minute plus a flat fee of $5.99,
- and a description of a situation in which the *y*-intercept of its graph is $25.

51. Which equation does *not* have a *y*-intercept of 5?

Ⓐ $2x = y - 5$ Ⓑ $3x + y = 5$

Ⓒ $y = x + 5$ Ⓓ $2x - y = 5$

52. Which situation below is modeled by the graph?

Ⓐ You have $100 and plan to spend $5 each week.

Ⓑ You have $100 and plan to save $5 each week.

Ⓒ You need $100 for a new CD player and plan to save $5 each week.

Ⓓ You need $100 for a new CD player and plan to spend $5 each week.
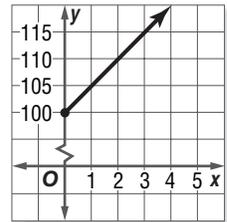
Extending the Lesson

53. The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers, $A \geq 0$, and A and B are not both zero. Solve $Ax + By = C$ for y. *Your answer is written in slope-intercept form.*

54. Use the slope-intercept equation in Exercise 53 to write expressions for the slope and *y*-intercept in terms of A, B, and C.

55. Use the expressions in Exercise 54 to find the slope and *y*-intercept of each equation.

a. $2x + y = -4$ **b.** $3x + 4y = 12$ **c.** $2x - 3y = 9$

Maintain Your Skills

Mixed Review   Write a direct variation equation that relates *x* and *y*. Assume that *y* varies directly as *x*. Then solve.   *(Lesson 5-2)*

56. If $y = 45$ when $x = 60$, find x when $y = 8$.

57. If $y = 15$ when $x = 4$, find y when $x = 10$.

Find the slope of the line that passes through each pair of points.   *(Lesson 5-1)*

58. $(-3, 0), (-4, 6)$ **59.** $(3, -1), (3, -4)$ **60.** $(5, -5), (9, 2)$

61. Write the numbers 2.5, $\frac{3}{4}$, -0.5, $\frac{7}{8}$ in order from least to greatest.   *(Lesson 2-7)*

Solve each equation.   *(Lesson 1-3)*
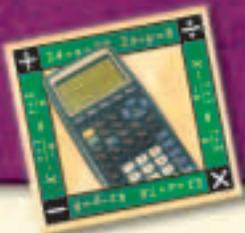
62. $x = \dfrac{15 - 9}{2}$ **63.** $3(7) + 2 = b$ **64.** $q = 6^2 - 2^2$

Getting Ready for the Next Lesson

PREREQUISITE SKILL   Find the slope of the line that passes through each pair of points.   *(To review **slope**, see Lesson 5-1.)*

65. $(-1, 2), (1, -2)$ **66.** $(5, 8), (-2, 8)$ **67.** $(1, -1), (10, -13)$

CONTENTS

Graphing Calculator

Families of Linear Graphs

A family of people is a group of people related by birth, marriage, or adoption. Recall that a *family of graphs* includes graphs and equations of graphs that have at least one characteristic in common.

Families of linear graphs fall into two categories—those with the same slope and those with the same y-intercept. A graphing calculator is a useful tool for studying a group of graphs to determine whether they form a family.
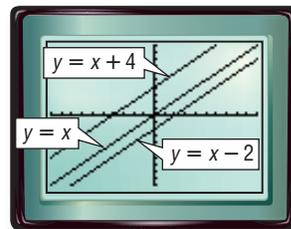
Example 1

Graph $y = x$, $y = x + 4$, and $y = x - 2$ in the standard viewing window. Describe any similarities and differences among the graphs. Write a description of the family.

Enter the equations in the Y= list as Y1, Y2, and Y3. Then graph the equations.

KEYSTROKES: *Review graphing on pages 224 and 225.*

$y = x + 4$
$y = x$
$y = x - 2$

$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

- The graph of $y = x$ has a slope of 1 and a y-intercept of 0.
- The graph of $y = x + 4$ has a slope of 1 and a y-intercept of 4.
- The graph of $y = x - 2$ has a slope of 1 and a y-intercept of -2.

Notice that the graph of $y = x + 4$ is the same as the graph of $y = x$, moved 4 units up. Also, the graph of $y = x - 2$ is the same as the graph of $y = x$, moved 2 units down. All graphs have the same slope and different intercepts.
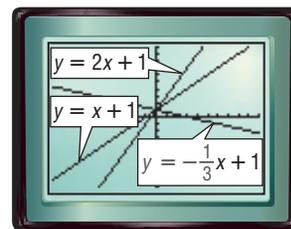
Because they all have the same slope, this family of graphs can be described as linear graphs with a slope of 1.

Example 2

Graph $y = x + 1$, $y = 2x + 1$, and $y = -\frac{1}{3}x + 1$ in the standard viewing window. Describe any similarities and differences among the graphs. Write a description of the family.

Enter the equations in the Y= list and graph.

$y = 2x + 1$
$y = x + 1$
$y = -\frac{1}{3}x + 1$

$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

- The graph of $y = x + 1$ has a slope of 1 and a y-intercept of 1.
- The graph of $y = 2x + 1$ has a slope of 2 and a y-intercept of 1.
- The graph of $y = -\frac{1}{3}x + 1$ has a slope of $-\frac{1}{3}$ and a y-intercept of 1.

These graphs have the same intercept and different slopes. This family of graphs can be described as linear graphs with a y-intercept of 1.

Investigation

Nonlinear functions can also be defined in terms of a family of graphs. Consider the **absolute value function** $y = |x|$, where $y \geq 0$ for all values of x.

Example 3

Graph $y = |x|$, $y = |x| + 2$, and $y = |x + 3|$ in the standard viewing window. Describe any similarities or differences among the graphs.

Enter the equations in the Y= list and graph.

KEYSTROKES: $\boxed{\text{Y=}}$ $\boxed{\text{MATH}}$ $\boxed{\blacktriangleright}$ 1 $\boxed{\text{X,T,}\theta\text{,}n}$ $\boxed{\text{ENTER}}$

$\boxed{\text{MATH}}$ $\boxed{\blacktriangleright}$ 1 $\boxed{\text{X,T,}\theta\text{,}n}$ $\boxed{\text{)}}$ $\boxed{+}$ 2 $\boxed{\text{ENTER}}$

$\boxed{\text{MATH}}$ $\boxed{\blacktriangleright}$ 1 $\boxed{\text{X,T,}\theta\text{,}n}$ $\boxed{+}$ 3 $\boxed{\text{)}}$ $\boxed{\text{ZOOM}}$ 6

$y = |x| + 2$
$y = |x + 3|$
$y = |x|$
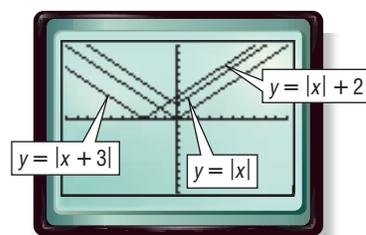
$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

- The graph of $y = |x|$ is v-shaped with its vertex at the origin.
- The graph of $y = |x| + 2$ is v-shaped with its vertex at $(0, 2)$.
- The graph of $y = |x + 3|$ is v-shaped with its vertex at $(-3, 0)$.

These graphs have the same shape, but they are positioned in different places on the coordinate plane.

Exercises

Graph each set of equations on the same screen. Describe any similarities or differences among the graphs. If the graphs are part of the same family, describe the family.

1. $y = -4$
$y = 0$
$y = 7$

2. $y = -x + 1$
$y = 2x + 1$
$y = \frac{1}{4}x + 1$

3. $y = x + 4$
$y = 2x + 4$
$y = 2x - 4$

4. $y = \frac{1}{2}x + 2$
$y = \frac{1}{3}x + 3$
$y = \frac{1}{4}x + 4$

5. $y = -2x - 2$
$y = 2x - 2$
$y = \frac{1}{2}x - 2$

6. $y = 3x$
$y = 3x + 6$
$y = 3x - 7$

7. $y = |x|$
$y = -3|x|$
$y = |-3x|$

8. $y = |x|$
$y = |x| + 3$
$y = |x| - 2$

9. $y = |x|$
$y = |2x| + 4$
$y = 3|x| - 5$

10. MAKE A CONJECTURE Write a paragraph explaining how the values of m and b in the slope-intercept form affect the graph of a linear equation.

11. Families of graphs are also called **classes of functions**. Describe the similarities and differences in the class of functions $f(x) = x + c$, where c is any real number.

12. Describe the similarities and differences in the classes of functions $f(x) = |x| + c$, and $f(x) = |x + c|$, where c is any real number.

CONTENTS

Writing Equations in Slope-Intercept Form

What You'll Learn

- Write an equation of a line given the slope and one point on a line.
- Write an equation of a line given two points on the line.
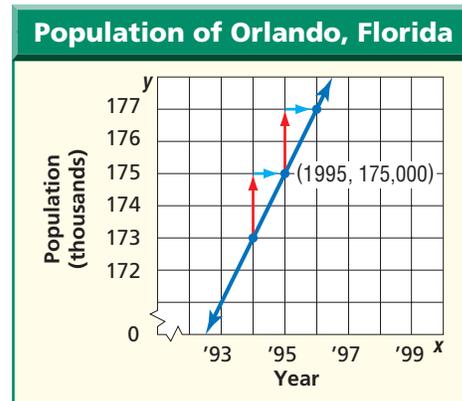
Vocabulary

- linear extrapolation

How can slope-intercept form be used to make predictions?

In 1995, the population of Orlando, Florida, was about 175,000. At that time, the population was growing at a rate of about 2000 per year.

x (year)	y (population)
⋮	⋮
1994	173,000
1995	175,000
1996	177,000
⋮	⋮

Population of Orlando, Florida

If you could write an equation based on the slope, 2000, and the point (1995, 175,000), you could predict the population for another year.

WRITE AN EQUATION GIVEN THE SLOPE AND ONE POINT You have learned how to write an equation of a line when you know the slope and a specific point, the y-intercept. The following example shows how to write an equation when you know the slope and any point on the line.

Example 1 Write an Equation Given Slope and One Point

Write an equation of a line that passes through (1, 5) with slope 2.

Step 1 The line has slope 2. To find the y-intercept, replace m with 2 and (x, y) with (1, 5) in the slope-intercept form. Then, solve for b.

$$y = mx + b \qquad \text{Slope-intercept form}$$
$$5 = 2(1) + b \qquad \text{Replace } m \text{ with 2, } y \text{ with 5, and } x \text{ with 1.}$$
$$5 = 2 + b \qquad \text{Multiply.}$$
$$5 - 2 = 2 + b - 2 \qquad \text{Subtract 2 from each side.}$$
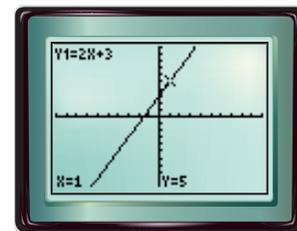$$3 = b \qquad \text{Simplify.}$$

Step 2 Write the slope-intercept form using $m = 2$ and $b = 3$.

$$y = mx + b \qquad \text{Slope-intercept form}$$
$$y = 2x + 3 \qquad \text{Replace } m \text{ with 2 and } b \text{ with 3.}$$

Therefore, the equation is $y = 2x + 3$.

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CHECK You can check your result by graphing
$y = 2x + 3$ on a graphing calculator.
Use the **CALC** menu to verify that it
passes through (1, 5).

$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

WRITE AN EQUATION GIVEN TWO POINTS Sometimes you do not
know the slope of a line, but you know two points on the line. In this case, find
the slope of the line. Then follow the steps in Example 1.

Example **2** *Write an Equation Given Two Points*

Multiple-Choice Test Item

The table of ordered pairs shows the coordinates of the two points
on the graph of a function. Which equation describes the function?

Ⓐ $y = -\frac{1}{3}x - 2$ Ⓑ $y = 3x - 2$

Ⓒ $y = -\frac{1}{3}x + 2$ Ⓓ $y = \frac{1}{3}x - 2$

x	y
−3	−1
6	−4

Read the Test Item
The table represents the ordered pairs $(-3, -1)$ and $(6, -4)$.

Solve the Test Item

Step 1 Find the slope of the line containing the points. Let $(x_1, y_1) = (-3, -1)$ and
$(x_2, y_2) = (6, -4)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \dfrac{-4 - (-1)}{6 - (-3)}$ $x_1 = -3, x_2 = 6, y_1 = -1, y_2 = -4$

$m = \dfrac{-3}{9}$ or $-\dfrac{1}{3}$ Simplify.

Step 2 You know the slope and two points. Choose one point and find the
y-intercept. In this case, we chose $(6, -4)$.

$y = mx + b$ Slope-intercept form

$-4 = -\frac{1}{3}(6) + b$ Replace m with $-\frac{1}{3}$, x with 6, and y with -4.

$-4 = -2 + b$ Multiply.

$-4 + 2 = -2 + b + 2$ Add 2 to each side.

$-2 = b$ Simplify.

Step 3 Write the slope-intercept form using $m = -\frac{1}{3}$ and $b = -2$.

$y = mx + b$ Slope-intercept form

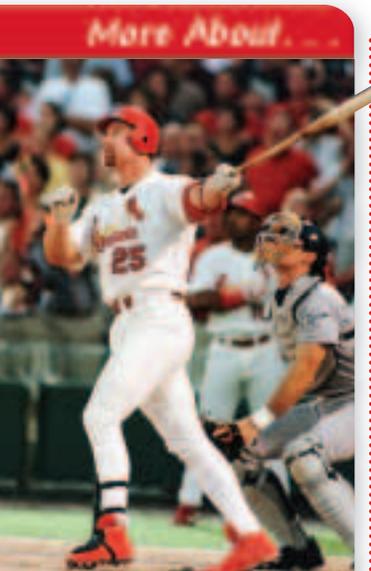$y = -\frac{1}{3}x - 2$ Replace m with $-\frac{1}{3}$ and b with -2.

Therefore, the equation is $y = -\frac{1}{3}x - 2$. The answer is A.

Test-Taking Tip
You can check your result
by graphing. The line
should pass through
$(-3, -1)$ and $(6, -4)$.

www.algebra1.com/extra_examples

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Example 3 Write an Equation to Solve a Problem

BASEBALL In the middle of the 1998 baseball season, Mark McGwire seemed to be on track to break the record for most runs batted in. After 40 games, McGwire had 45 runs batted in. After 86 games, he had 87 runs batted in. Write a linear equation to estimate the number of runs batted in for any number of games that season.

Explore You know the number of runs batted in after 40 and 86 games.

Plan Let x represent the number of games. Let y represent the number of runs batted in. Write an equation of the line that passes through (40, 45) and (86, 87).

Runs Batted In
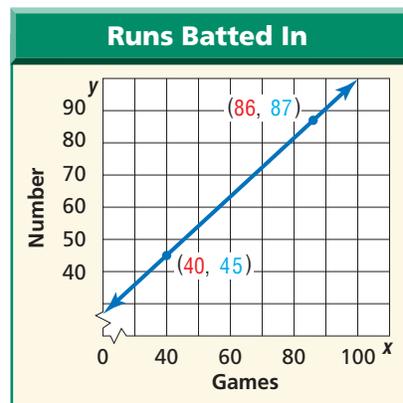
Solve Find the slope.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \dfrac{87 - 45}{86 - 40}$ Let $(x_1, y_1) = (40, 45)$ and $(x_2, y_2) = (86, 87)$.

$m = \dfrac{42}{46}$ or about 0.91   Simplify.

Choose (40, 45) and find the y-intercept of the line.

$y = mx + b$ Slope-intercept form

$45 = 0.91(40) + b$ Replace m with 0.91, x with 40, and y with 45.

$45 = 36.4 + b$ Multiply.

$45 - 36.4 = 36.4 + b - 36.4$ Subtract 36.4 from each side.

$8.6 = b$ Simplify.

Write the slope-intercept form using $m = 0.91$, and $b = 8.6$.

$y = mx + b$ Slope-intercept form

$y = 0.91x + 8.6$ Replace m with 0.91 and b with 8.6.

Therefore, the equation is $y = 0.91x + 8.6$.

Examine Check your result by substituting the coordinates of the point not chosen, (86, 87), into the equation.

$y = 0.91x + 8.6$ Original equation

$87 \stackrel{?}{=} 0.91(86) + 8.6$ Replace y with 87 and x with 86.

$87 \stackrel{?}{=} 78.26 + 8.6$ Multiply.

$87 \approx 86.86 \checkmark$ The slope was rounded, so the answers vary slightly.

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Concept Summary
Writing Equations

Given the Slope and One Point	Given Two Points
Step 1 Substitute the values of *m, x,* and *y* into the slope-intercept form and solve for *b*.	**Step 1** Find the slope.
	Step 2 Choose one of the two points to use.
Step 2 Write the slope-intercept form using the values of *m* and *b*.	**Step 3** Then, follow the steps for writing an equation given the slope and one point.

When you use a linear equation to predict values that are beyond the range of the data, you are using **linear extrapolation**.

Example 4 Linear Extrapolation

SPORTS   The record for most runs batted in during a single season is 190. Use the equation in Example 3 to decide whether a baseball fan following the 1998 season would have expected McGwire to break the record in the 162 games played that year.

$y = 0.91x + 8.6$ Original equation

$y = 0.91(162) + 8.6$ Replace *x* with 162.

$y \approx 156$ Simplify.

Since the record is 190 runs batted in, a fan would have predicted that Mark McGwire would not break the record.

Be cautious when making a prediction using just two given points. The model may be *approximately* correct, but still give inaccurate predictions. For example, in 1998, Mark McGwire had 147 runs batted in, which was nine less than the prediction.

Check for Understanding

Concept Check

1. **Compare and contrast** the process used to write an equation given the slope and one point with the process used for two points.

2. **OPEN ENDED**   Write an equation in slope-intercept form of a line that has a *y*-intercept of 3.

3. **Tell** whether the statement is *sometimes, always,* or *never* true. Explain. *You can write the equation of a line given its x- and y-intercepts.*

Guided Practice

Write an equation of the line that passes through each point with the given slope.

4. $(4, -2)$, $m = 2$ 5. $(3, 7)$, $m = -3$ 6. $(-3, 5)$, $m = -1$

Write an equation of the line that passes through each pair of points.

7. $(5, 1)$, $(8, -2)$ 8. $(6, 0)$, $(0, 4)$ 9. $(5, 2)$, $(-7, -4)$

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

10. The table of ordered pairs shows the coordinates of the two points on the graph of a line. Which equation describes the line?

x	y
−5	2
0	7

Ⓐ $y = x + 7$ Ⓑ $y = x - 7$
Ⓒ $y = -5x + 2$ Ⓓ $y = 5x + 2$

◀ **CONTENTS** ▶

Write an equation of the line that passes through each point with the given slope.

11.

$(1, 2)$

$m = 3$

12.

$m = -1$

$(4, -1)$
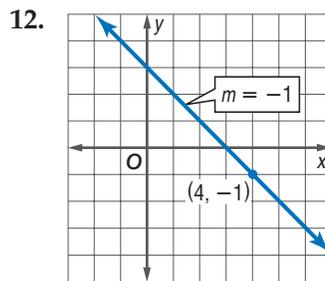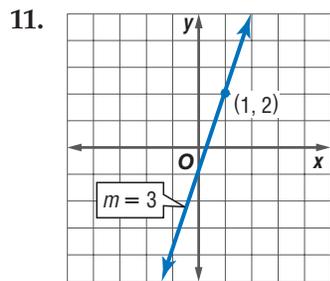
13. $(5, -2)$, $m = 3$

14. $(5, 4)$, $m = -5$

15. $(3, 0)$, $m = -2$

16. $(5, 3)$, $m = \dfrac{1}{2}$

17. $(-3, -1)$, $m = -\dfrac{2}{3}$

18. $(-3, -5)$, $m = -\dfrac{5}{3}$

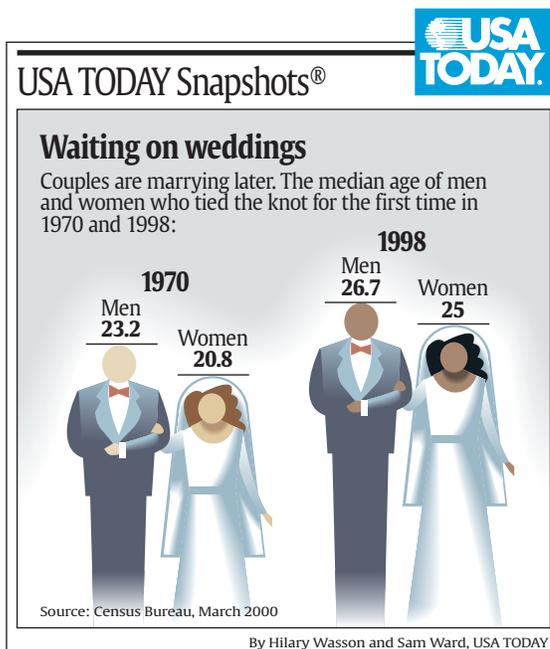**Write an equation of the line that passes through each pair of points.**

19.

$(5, 2)$

$(4, 1)$

20.

$(0, 2)$

$(2, 0)$
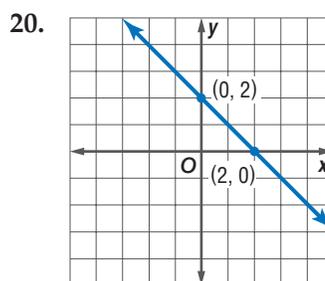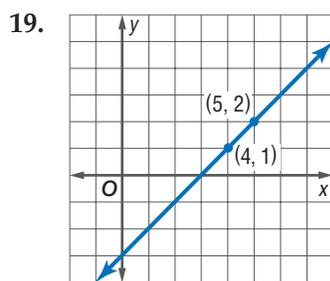
21. $(4, 2)$, $(-2, -4)$

22. $(3, -2)$, $(6, 4)$

23. $(-1, 3)$, $(2, -3)$

24. $(2, -2)$, $(3, 2)$

25. $(7, -2)$, $(-4, -2)$

26. $(0, 5)$, $(-3, 5)$

27. $(1, 1)$, $(7, 4)$

28. $(5, 7)$, $(0, 6)$

29. $\left(-\dfrac{5}{4}, 1\right), \left(-\dfrac{1}{4}, \dfrac{3}{4}\right)$

Write an equation of the line that has each pair of intercepts.

30. x-intercept: -3, y-intercept: 5

31. x-intercept: 3, y-intercept: 4

32. x-intercept: 6, y-intercept: 3

33. x-intercept: 2, y-intercept: -2

MARRIAGE AGE **For Exercises 34–37, use the information in the graphic.**

34. Write a linear equation to predict the median age that men marry M for any year t.

35. Use the equation to predict the median age of men who marry for the first time in 2005.

36. Write a linear equation to predict the median age that women marry W for any year t.

37. Use the equation to predict the median age of women who marry for the first time in 2005.

USA TODAY Snapshots®

Waiting on weddings

Couples are marrying later. The median age of men and women who tied the knot for the first time in 1970 and 1998:

1970
Men **23.2**
Women **20.8**

1998
Men **26.7**
Women **25**

Source: Census Bureau, March 2000

By Hilary Wasson and Sam Ward, USA TODAY

POPULATION For Exercises 38 and 39, use the data at the top of page 280.

38. Write a linear equation to find Orlando's population for any year.

39. Predict what Orlando's population will be in 2010.

40. CANOE RENTAL If you rent a canoe for 3 hours, you will pay $45. Write a linear equation to find the total cost C of renting the canoe for h hours.

CANOE RENTALS
DAILY RATE PLUS
$10 PER HOUR

For Exercises 41–43, consider line ℓ that passes through (14, 2) and (28, 6).

41. Write an equation for line ℓ.

42. What is the slope of line ℓ?

43. Where does line ℓ intersect the x-axis? the y-axis?

44. CRITICAL THINKING The x-intercept of a line is p, and the y-intercept is q. Write an equation of the line.

45. | WRITING IN MATH | Answer the question that was posed at the beginning of the lesson.

How can slope-intercept form be used to make predictions?

Include the following in your answer:

• a definition of linear extrapolation, and

• an explanation of how slope-intercept form is used in linear extrapolation.

46. Which is an equation for the line with slope $\frac{1}{3}$ through $(-2, 1)$?

Ⓐ $y = \frac{1}{3}x + 1$ Ⓑ $y = \frac{1}{3}x + \frac{5}{3}$ Ⓒ $y = \frac{1}{3}x - \frac{5}{3}$ Ⓓ $y = \frac{1}{3}x + \frac{1}{3}$

47. About 20,000 fewer babies were born in California in 1996 than in 1995. In 1995, about 560,000 babies were born. Which equation can be used to predict the number of babies y (in thousands), born x years after 1995?

Ⓐ $y = 20x + 560$ Ⓑ $y = -20x + 560$

Ⓒ $y = -20x - 560$ Ⓓ $y = 20x - 560$

Maintain Your Skills

Mixed Review   **Graph each equation.** *(Lesson 5-3)*

48. $y = 3x - 2$ **49.** $x + y = 6$ **50.** $x + 2y = 8$

51. HEALTH Each time your heart beats, it pumps 2.5 ounces of blood through your heart. Write a direct variation equation that relates the total volume of blood V with the number of times your heart beats b. *(Lesson 5-2)*

State the domain of each relation. *(Lesson 4-3)*

52. $\{(0, 8), (9, -2), (4, 2)\}$ **53.** $\{(-2, 1), (5, 1), (-2, 7), (0, -3)\}$

Replace each ● with $<$, $>$, or $=$ to make a true sentence. *(Lesson 2-7)*

54. -3 ● -5 **55.** 4 ● $\frac{16}{3}$ **56.** $\frac{3}{4}$ ● $\frac{2}{3}$

Getting Ready for the Next Lesson   **PREREQUISITE SKILL   Find each difference.**
*(To review **subtracting integers**, see Lesson 2-2.)*

57. $4 - 7$ **58.** $5 - 12$ **59.** $2 - (-3)$

60. $-1 - 4$ **61.** $-7 - 8$ **62.** $-5 - (-2)$

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Writing Equations in Point-Slope Form
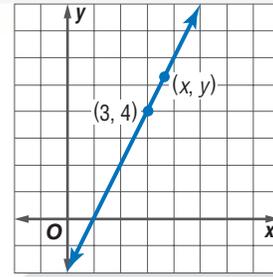
What **You'll Learn**

- Write the equation of a line in point-slope form.
- Write linear equations in different forms.

Vocabulary

- point-slope form

How **can you use the slope formula to write an equation of a line?**

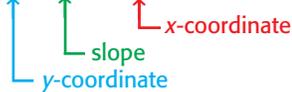The graph shows a line with slope 2 that passes through $(3, 4)$. Another point on the line is (x, y).

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$2 = \dfrac{y - 4}{x - 3}$ $(x_2, y_2) = (x, y)$
$(x_1, y_1) = (3, 4)$

$2(x - 3) = \dfrac{y - 4}{x - 3}(x - 3)$ Multiply each side by $(x - 3)$.

$2(x - 3) = y - 4$ Simplify.

$y - 4 = 2(x - 3)$ Symmetric Property of Equality

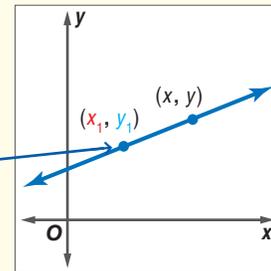↳ x-coordinate
↳ slope
↳ y-coordinate

POINT-SLOPE FORM   The equation above was generated using the coordinates of a known point and the slope of the line. It is written in **point-slope form**.

Key Concept   *Point–Slope Form*

- **Words**   The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line.

- **Model**

- **Symbols** $y - y_1 = m(x - x_1)$ given point

Example 1   **Write an Equation Given Slope and a Point**

Study Tip

Point–Slope Form

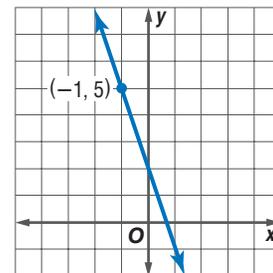Remember, (x_1, y_1) represents the *given* point, and (x, y) represents *any other* point on the line.

Write the point-slope form of an equation for a line that passes through $(-1, 5)$ with slope -3.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 5 = -3[x - (-1)]$ $(x_1, y_1) = (-1, 5)$

$y - 5 = -3(x + 1)$ Simplify.

Therefore, the equation is $y - 5 = -3(x + 1)$.

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Vertical lines cannot be written in point-slope form because the slope is undefined. However, since the slope of a horizontal line is 0, horizontal lines can be written in point-slope form.
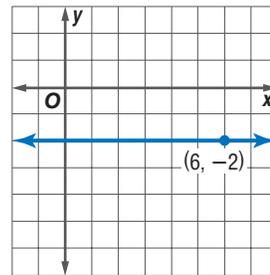
Example 2 Write an Equation of a Horizontal Line

Write the point-slope form of an equation for a horizontal line that passes through $(6, -2)$.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-2) = 0(x - 6)$ $(x_1, y_1) = (6, -2)$

$y + 2 = 0$ Simplify.

Therefore, the equation is $y + 2 = 0$.

FORMS OF LINEAR EQUATIONS   You have learned about three of the most common forms of linear equations.

Concept Summary   *Forms of Linear Equations*

Form	Equation	Description
Slope-Intercept	$y = mx + b$	m is the slope, and b is the y-intercept.
Point-Slope	$y - y_1 = m(x - x_1)$	m is the slope and (x_1, y_1) is a given point.
Standard	$Ax + By = C$	A and B are not both zero. Usually A is nonnegative and A, B, and C are integers whose greatest common factor is 1.

Linear equations in point-slope form can be written in slope-intercept or standard form.

Example 3 Write an Equation in Standard Form

Write $y + 5 = -\frac{5}{4}(x - 2)$ in standard form.

In standard form, the variables are on the left side of the equation. A, B, and C are all integers.

$y + 5 = -\dfrac{5}{4}(x - 2)$ Original equation

$4(y + 5) = 4\left(-\dfrac{5}{4}\right)(x - 2)$ Multiply each side by 4 to eliminate the fraction.

$4y + 20 = -5(x - 2)$ Distributive Property

$4y + 20 = -5x + 10$ Distributive Property

$4y + 20 - 20 = -5x + 10 - 20$ Subtract 20 from each side.

$4y = -5x - 10$ Simplify.

$4y + 5x = -5x - 10 + 5x$ Add $5x$ to each side.

$5x + 4y = -10$ Simplify.

The standard form of the equation is $5x + 4y = -10$.

Example 4 Write an Equation in Slope-Intercept Form

Write $y - 2 = \frac{1}{2}(x + 5)$ in slope-intercept form.

In slope-intercept form, y is on the left side of the equation. The constant and x are on the right side.

$$y - 2 = \frac{1}{2}(x + 5) \qquad \text{Original equation}$$

$$y - 2 = \frac{1}{2}x + \frac{5}{2} \qquad \text{Distributive Property}$$

$$y - 2 + 2 = \frac{1}{2}x + \frac{5}{2} + 2 \qquad \text{Add 2 to each side.}$$

$$y = \frac{1}{2}x + \frac{9}{2} \qquad 2 = \frac{4}{2} \text{ and } \frac{4}{2} + \frac{5}{2} = \frac{9}{2}$$

The slope-intercept form of the equation is $y = \frac{1}{2}x + \frac{9}{2}$.

You can draw geometric figures on a coordinate plane and use the point-slope form to write equations of the lines.

Example 5 Write an Equation in Point-Slope Form

GEOMETRY The figure shows right triangle ABC.

a. **Write the point-slope form of the line containing the hypotenuse $\overline{AB}$.**

Step 1 First, find the slope of $\overline{AB}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

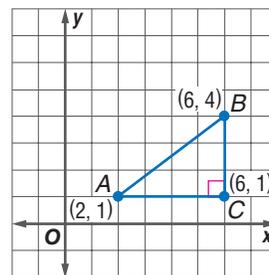$$= \frac{4 - 1}{6 - 2} \text{ or } \frac{3}{4} \qquad (x_1, y_1) = (2, 1), (x_2, y_2) = (6, 4)$$

Step 2 You can use either point for (x_1, y_1) in the point-slope form.

Method 1 Use (6, 4).

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{3}{4}(x - 6)$$

Method 2 Use (2, 1).

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{3}{4}(x - 2)$$

b. **Write each equation in standard form.**

$y - 4 = \frac{3}{4}(x - 6)$	Original equation
$4(y - 4) = 4\left(\frac{3}{4}\right)(x - 6)$	Multiply each side by 4.
$4y - 16 = 3(x - 6)$	Multiply.
$4y - 16 = 3x - 18$	Distributive Property
$4y = 3x - 2$	Add to each side.
$-3x + 4y = -2$	Subtract $3x$ from each side.
$3x - 4y = 2$	Multiply each side by -1.

$$y - 1 = \frac{3}{4}(x - 2)$$
$$4(y - 1) = 4\left(\frac{3}{4}\right)(x - 2)$$
$$4y - 4 = 3(x - 2)$$
$$4y - 4 = 3x - 6$$
$$4y = 3x - 2$$
$$-3x + 4y = -2$$
$$3x - 4y = 2$$

Regardless of which point was used to find the point-slope form, the standard form results in the same equation.

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Concept Check

1. **Explain** what x_1 and y_1 in the point-slope form of an equation represent.

2. **FIND THE ERROR** Tanya and Akira wrote the point-slope form of an equation for a line that passes through $(-2, -6)$ and $(1, 6)$. Tanya says that Akira's equation is wrong. Akira says they are both correct. Who is correct? Explain.
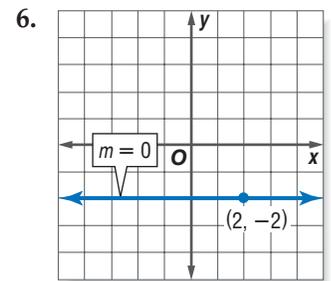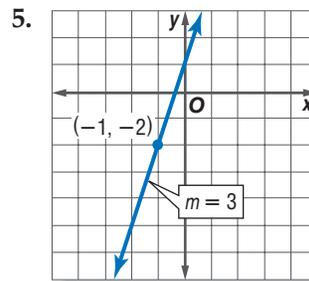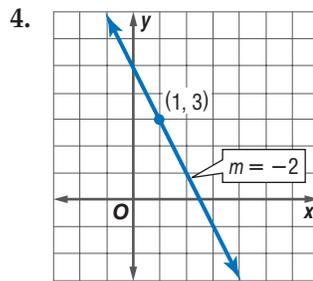
Tanya	Akira
$y + 6 = 4(x + 2)$	$y - 6 = 4(x - 1)$

3. **OPEN ENDED** Write an equation in point-slope form. Then write an equation for the same line in slope-intercept form.

Guided Practice **Write the point-slope form of an equation for a line that passes through each point with the given slope.**

4.

5.

6.

Write each equation in standard form.

7. $y - 5 = 4(x + 2)$

8. $y + 3 = -\dfrac{3}{4}(x - 1)$

9. $y - 3 = 2.5(x + 1)$

Write each equation in slope-intercept form.
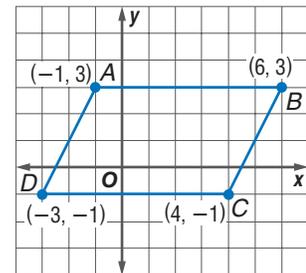
10. $y + 6 = 2(x - 2)$

11. $y + 3 = -\dfrac{2}{3}(x - 6)$

12. $y - \dfrac{7}{2} = \dfrac{1}{2}(x - 4)$

Application **GEOMETRY For Exercises 13 and 14, use parallelogram $ABCD$.**
A parallelogram has opposite sides parallel.

13. Write the point-slope form of the line containing $\overline{AD}$.

14. Write the standard form of the line containing $\overline{AD}$.

Homework Help

For Exercises	See Examples
15–26	1
27–28	2
29–40	3
41–52	4

Extra Practice
See page 832.

Write the point-slope form of an equation for a line that passes through each point with the given slope.

15. $(3, 8)$, $m = 2$

16. $(-4, -3)$, $m = 1$

17. $(-2, 4)$, $m = -3$

18. $(-6, 1)$, $m = -4$

19. $(-3, 6)$, $m = 0$

20. $(9, 1)$, $m = \dfrac{2}{3}$

21. $(8, -3)$, $m = \dfrac{3}{4}$

22. $(-6, 3)$, $m = -\dfrac{2}{3}$

23. $(1, -3)$, $m = -\dfrac{5}{8}$

24. $(9, -5)$, $m = 0$

25. $(-4, 8)$, $m = \dfrac{7}{2}$

26. $(1, -4)$, $m = -\dfrac{8}{3}$

www.algebra1.com/self_check_quiz

CONTENTS

27. Write the point-slope form of an equation for a horizontal line that passes through $(5, -9)$.

28. A horizontal line passes through $(0, 7)$. Write the point-slope form of its equation.

Write each equation in standard form.

29. $y - 13 = 4(x - 2)$ **30.** $y + 3 = 3(x + 5)$ **31.** $y - 5 = -2(x + 6)$

32. $y + 3 = -5(x + 1)$ **33.** $y + 7 = \frac{1}{2}(x + 2)$ **34.** $y - 1 = \frac{5}{6}(x - 4)$

35. $y - 2 = -\frac{2}{5}(x - 8)$ **36.** $y + 4 = -\frac{1}{3}(x - 12)$ **37.** $y + 2 = \frac{5}{3}(x + 6)$

38. $y + 6 = \frac{3}{2}(x - 4)$ **39.** $y - 6 = 1.3(x + 7)$ **40.** $y - 2 = -2.5(x - 1)$

Write each equation in slope-intercept form.

41. $y - 2 = 3(x - 1)$ **42.** $y - 5 = 6(x + 1)$ **43.** $y + 2 = -2(x - 5)$

44. $y - 1 = -7(x - 3)$ **45.** $y + 3 = \frac{1}{2}(x + 4)$ **46.** $y - 1 = \frac{2}{3}(x + 9)$

47. $y + 3 = -\frac{1}{4}(x + 2)$ **48.** $y - 5 = -\frac{2}{5}(x + 15)$ **49.** $y + \frac{1}{2} = x - \frac{1}{2}$

50. $y - \frac{1}{3} = -2\left(x + \frac{1}{3}\right)$ **51.** $y + \frac{1}{4} = -3\left(x + \frac{1}{2}\right)$ **52.** $y + \frac{3}{5} = -4\left(x - \frac{1}{2}\right)$

53. Write the point-slope form, slope-intercept form, and standard form of an equation for a line that passes through $(5, -3)$ with slope 10.

54. Line ℓ passes through $(1, -6)$ with slope $\frac{3}{2}$. Write the point-slope form, slope-intercept form, and standard form of an equation for line ℓ.

BUSINESS **For Exercises 55–57, use the following information.**
A home security company provides security systems for $5 per week, plus an installation fee. The total fee for 12 weeks of service is $210.

55. Write the point-slope form of an equation to find the total fee y for any number of weeks x.

56. Write the equation in slope-intercept form.

57. What is the flat fee for installation?

MOVIES **For Exercises 58–60, use the following information.**
Between 1990 and 1999, the number of movie screens in the United States increased by about 1500 each year. In 1996, there were 29,690 movie screens.

58. Write the point-slope form of an equation to find the total number of screens y for any year x.

59. Write the equation in slope-intercept form.

60. Predict the number of movie screens in the United States in 2005.
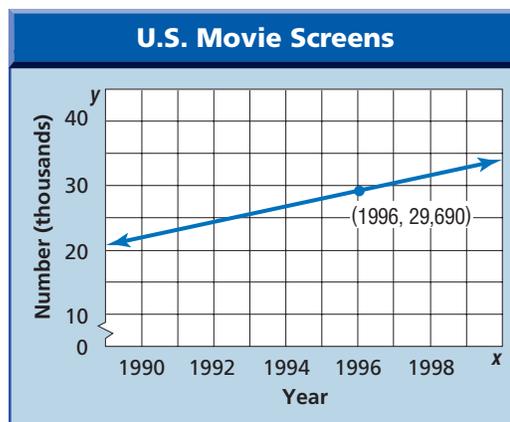
U.S. Movie Screens

Source: Motion Picture Association of America

Online Research **Data Update** What has happened to the number of movie screens since 1999? Visit www.algebra1.com/data_update to learn more.

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GEOMETRY For Exercises 61–63, use square *PQRS*.
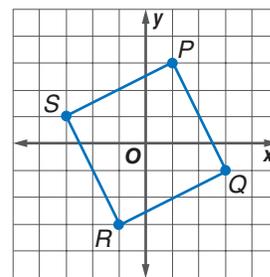
61. Write a point-slope equation of the line containing each side.

62. Write the slope-intercept form of each equation.

63. Write the standard form of each equation.

64. CRITICAL THINKING A line contains the points (9, 1) and (5, 5). Write a convincing argument that the same line intersects the *x*-axis at (10, 0).

65. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can you use the slope formula to write an equation of a line?

Include the following in your answer:

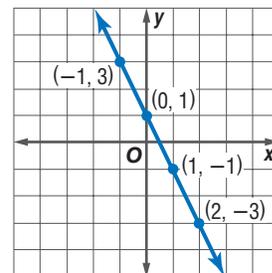- an explanation of how you can use the slope formula to write the point-slope form.

66. Which equation represents a line that *neither* passes through (0, 1) *nor* has a slope of 3?

Ⓐ $-2x + y = 1$ Ⓑ $y + 1 = 3(x + 6)$

Ⓒ $y - 3 = 3(x - 6)$ Ⓓ $x - 3y = -15$

67. SHORT RESPONSE Write the slope-intercept form of an equation of a line that passes through (2, −5).

Extending the Lesson For Exercises 68–71, use the graph at the right.

68. Choose three different pairs of points from the graph. Write the slope-intercept form of the line using each pair.

69. Describe how the equations are related.

70. Choose a different pair of points from the graph and predict the equation of the line determined by these points. Check your conjecture by finding the equation.

71. MAKE A CONJECTURE What conclusion can you draw from this activity?

Maintain Your Skills

Mixed Review **Write the slope-intercept form of an equation of the line that satisfies each condition.** *(Lessons 5-3 and 5-4)*

72. slope −2 and *y*-intercept −5

73. passes through (−2, 4) with slope 3

74. passes through (2, −4) and (0, 6)

75. a horizontal line through (1, −1)

Solve each equation. *(Lesson 3-4)*

76. $4a - 5 = 15$

77. $7 + 3c = -11$

78. $\frac{2}{9}v - 6 = 14$

79. Evaluate $(25 - 4) \div (2^2 - 1^3)$. *(Lesson 1-3)*

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Write the multiplicative inverse of each number.
(For review of multiplicative inverses, see pages 800 and 801.)

80. 2

81. 10

82. 1

83. −1

84. $\frac{2}{3}$

85. $-\frac{1}{9}$

86. $\frac{5}{2}$

87. $-\frac{2}{3}$

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# Geometry: Parallel and Perpendicular Lines

What You'll Learn

- Write an equation of the line that passes through a given point, parallel to a given line.
- Write an equation of the line that passes through a given point, perpendicular to a given line.

Vocabulary

- parallel lines
- perpendicular lines

How can you determine whether two lines are parallel?

The graphing calculator screen shows a family of linear graphs whose slope is 1. Notice that the lines do not appear to intersect.
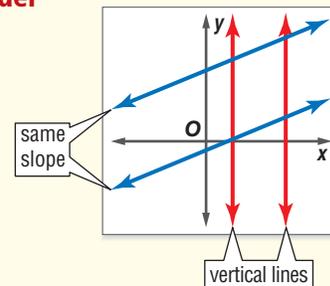
$y = x + 3$
$y = x$
$y = x - 3$
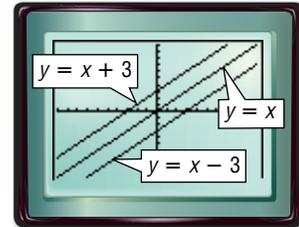
PARALLEL LINES Lines in the same plane that do not intersect are called **parallel lines**. Parallel lines have the same slope.

Key Concept — Parallel Lines in a Coordinate Plane

- **Words** Two nonvertical lines are parallel if they have the same slope. All vertical lines are parallel.

- **Model**

same slope

vertical lines

You can write the equation of a line parallel to a given line if you know a point on the line and an equation of the given line.

Example 1 Parallel Line Through a Given Point

Write the slope-intercept form of an equation for the line that passes through $(-1, -2)$ and is parallel to the graph of $y = -3x - 2$.

The line parallel to $y = -3x - 2$ has the same slope, -3. Replace m with -3, and (x_1, y_1) with $(-1, -2)$ in the point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-2) = -3[x - (-1)]$ Replace m with -3, y with -2, and x with -1.

$y + 2 = -3(x + 1)$ Simplify.

$y + 2 = -3x - 3$ Distributive Property

$y + 2 - 2 = -3x - 3 - 2$ Subtract 2 from each side.

$y = -3x - 5$ Write the equation in slope-intercept form.

Therefore, the equation is $y = -3x - 5$.

CONTENTS

CHECK You can check your result by graphing both equations. The lines appear to be parallel. The graph of $y = -3x - 5$ passes through $(-1, -2)$.

(−1, −2)

$y = -3x - 2$

$y = -3x - 5$
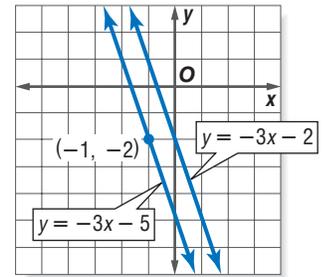
PERPENDICULAR LINES Lines that intersect at right angles are called **perpendicular lines**. There is a relationship between the slopes of perpendicular lines.
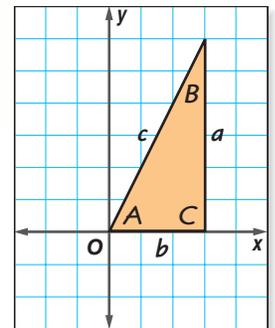
Algebra Activity

Perpendicular Lines

Model
- A scalene triangle is one in which no two sides are equal. Cut out a scalene right triangle ABC so that $\angle C$ is a right angle. Label the vertices and the sides as shown.
- Draw a coordinate plane on grid paper. Place $\triangle ABC$ on the coordinate plane so that A is at the origin and side b lies along the positive x-axis.

Analyze
1. Name the coordinates of B.
2. What is the slope of side c?
3. Rotate the triangle 90° counterclockwise so that A is still at the origin and side b is along the positive y-axis. Name the coordinates of B.
4. What is the slope of side c?
5. Repeat the activity for two other different scalene triangles.
6. For each triangle and its rotation, what is the relationship between the first position of side c and the second?
7. For each triangle and its rotation, describe the relationship between the coordinates of B in the first and second positions.
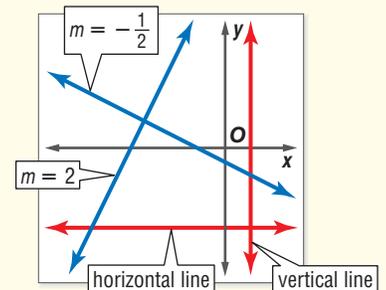8. Describe the relationship between the slopes of c in each position.

Make a Conjecture
9. Describe the relationship between the slopes of any two perpendicular lines.

Key Concept — Perpendicular Lines in a Coordinate Plane

- **Words** Two nonvertical lines are perpendicular if the product of their slopes is -1. That is, the slopes are *opposite reciprocals* of each other. Vertical lines and horizontal lines are also perpendicular.

- **Model**

  $m = -\dfrac{1}{2}$

  $m = 2$

  horizontal line

  vertical line

Lesson 5-6   Geometry: Parallel and Perpendicular Lines   **293**

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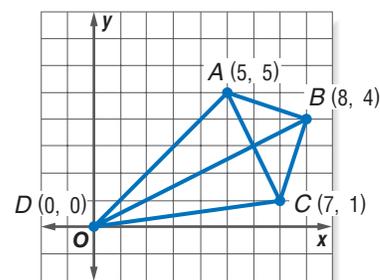
Example 2 Determine Whether Lines are Perpendicular

KITES The outline of a kite is shown on a coordinate plane. Determine whether $\overline{AC}$ is perpendicular to $\overline{BD}$.

Find the slope of each segment.

Slope of $\overline{AC}$: $m = \dfrac{5 - 1}{5 - 7}$ or -2

Slope of $\overline{BD}$: $m = \dfrac{4 - 0}{8 - 0}$ or $\dfrac{1}{2}$

The line segments are perpendicular because $\dfrac{1}{2}(-2) = -1$.

More About...

You can write the equation of a line perpendicular to a given line if you know a point on the line and the equation of the given line.

Example 3 Perpendicular Line Through a Given Point

Write the slope-intercept form for an equation of the line that passes through $(-3, -2)$ and is perpendicular to the graph of $x + 4y = 12$.

Step 1 Find the slope of the given line.

$x + 4y = 12$	Original equation
$x + 4y - x = 12 - x$	Subtract $1x$ from each side.
$4y = -1x + 12$	Simplify.
$\dfrac{4y}{4} = \dfrac{-1x + 12}{4}$	Divide each side by 4.
$y = -\dfrac{1}{4}x + 3$	Simplify.

Step 2 The slope of the given line is $-\dfrac{1}{4}$. So, the slope of the line perpendicular to this line is the opposite reciprocal of $-\dfrac{1}{4}$, or 4.

Step 3 Use the point-slope form to find the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = 4[x - (-3)]$	$(x_1, y_1) = (-3, -2)$ and $m = 4$
$y + 2 = 4(x + 3)$	Simplify.
$y + 2 = 4x + 12$	Distributive Property
$y + 2 - 2 = 4x + 12 - 2$	Subtract 2 from each side.
$y = 4x + 10$	Simplify.

Therefore, the equation of the line is $y = 4x + 10$.

CHECK Graph both equations on a graphing calculator. Use the **CALC** menu to verify that $y = 4x + 10$ passes through $(-3, -2)$.
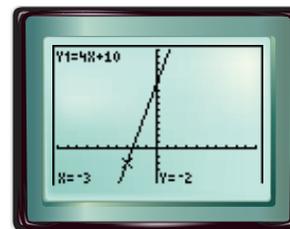
$[-15.16..., 15.16...]$ scl: 1 by $[-10, 10]$ scl: 1

Study Tip

Graphing Calculator
The lines will not appear to be perpendicular on a graphing calculator if the scales on the axes are not set correctly. After graphing, press ZOOM 5 to set the axes for a correct representation.

CONTENTS

Example 4 **Perpendicular Line Through a Given Point**

Write the slope-intercept form for an equation of a line perpendicular to the graph of $y = -\frac{1}{3}x + 2$ and passes through the *x*-intercept of that line.

Step 1 Find the slope of the perpendicular line. The slope of the given line is $-\frac{1}{3}$, therefore a perpendicular line has slope 3 because $-\frac{1}{3} \cdot 3 = -1$.

Step 2 Find the *x*-intercept of the given line.

$y = -\frac{1}{3}x + 2$	Original equation
$0 = -\frac{1}{3}x + 2$	Replace *y* with 0.
$-2 = -\frac{1}{3}x$	Subtract 2 from each side.
$6 = x$	Multiply each side by -3.

The *x*-intercept is at $(6, 0)$.

Step 3 Substitute the slope and the given point into the point-slope form of a linear equation. Then write the equation in slope-intercept form.

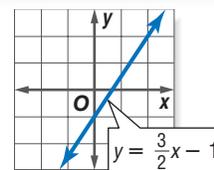|	
$y - y_1 = m(x - x_1)$	Point-slope form
$y - 0 = 3(x - 6)$	Replace *x* with 6, *y* with 0, and *m* with 3.
$y = 3x - 18$	Distributive Property

Check for Understanding

Concept Check
1. **Explain** how to find the slope of a line that is perpendicular to the line shown in the graph.
2. **OPEN ENDED** Give an example of two numbers that are negative reciprocals.
3. **Define** *parallel lines* and *perpendicular lines*.

$y = \frac{3}{2}x - 1$

Guided Practice
Write the slope-intercept form of an equation of the line that passes through the given point and is parallel to the graph of each equation.

4.

$y = -2x + 4$

$(0, -1)$

5.

$y = x + 5$

$(2, 3)$
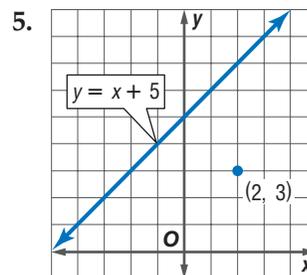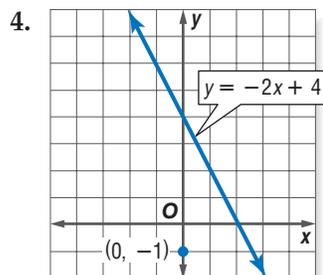
6. $(1, -3)$, $y = 2x - 1$

7. $(-2, 2)$, $-3x + y = 4$

8. **GEOMETRY** Quadrilateral $ABCD$ has vertices $A(-2, 1)$, $B(3, 3)$, $C(5, 7)$, and $D(0, 5)$. Determine whether $\overline{AC}$ is perpendicular to $\overline{BD}$.

Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation.

9. $(-3, 1)$, $y = \frac{1}{3}x + 2$
10. $(6, -2)$, $y = \frac{3}{5}x - 4$
11. $(2, -2)$, $2x + y = 5$

CONTENTS

Application   **12. GEOMETRY** The line with equation $y = 3x - 4$ contains side $\overline{AC}$ of right triangle ABC. If the vertex of the right angle C is at $(3, 5)$, what is an equation of the line that contains side $\overline{BC}$?

Practice and Apply

Write the slope-intercept form of an equation of the line that passes through the given point and is parallel to the graph of each equation.
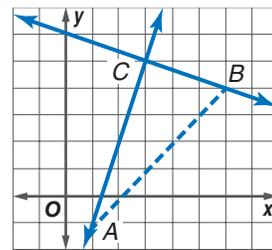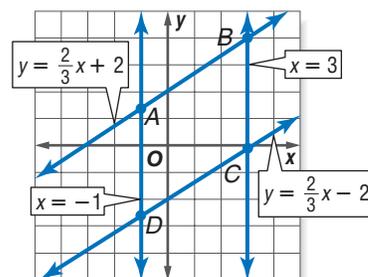
13. $(2, -7)$, $y = x - 2$

14. $(2, -1)$, $y = 2x + 2$

15. $(-3, 2)$, $y = x - 6$

16. $(4, -1)$, $y = 2x + 1$

17. $(-5, -4)$, $y = \frac{1}{2}x + 1$

18. $(3, 3)$, $y = \frac{2}{3}x - 1$

19. $(-4, -3)$, $y = -\frac{1}{3}x + 3$

20. $(-1, 2)$, $y = -\frac{1}{2}x - 4$

21. $(-3, 0)$, $2y = x - 1$

22. $(2, 2)$, $3y = -2x + 6$

23. $(-2, 3)$, $6x + y = 4$

24. $(2, 2)$, $3x - 4y = -4$

25. GEOMETRY A *parallelogram* is a quadrilateral in which opposite sides are parallel. Is $ABCD$ a parallelogram? Explain.

26. Write an equation of the line parallel to the graph of $y = 5x - 3$ and through the origin.

27. Write an equation of the line with y-intercept -6 that is parallel to the graph of $x - 3y = 8$.


Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation.

28. $(-2, 0)$, $y = x - 6$

29. $(1, 1)$, $y = 4x + 6$

30. $(-3, 1)$, $y = -3x + 7$

31. $(0, 5)$, $y = -8x + 4$

32. $(1, -3)$, $y = \frac{1}{2}x + 4$

33. $(4, 7)$, $y = \frac{2}{3}x - 1$

34. $(0, 4)$, $3x + 8y = 4$

35. $(-2, 7)$, $2x - 5y = 3$

36. $(6, -1)$, $3y + x = 3$

37. $(0, -1)$, $5x - y = 3$

38. $(8, -2)$, $5x - 7 = 3y$

39. $(3, -3)$, $3x + 7 = 2x$

40. Find an equation of the line that has a y-intercept of -2 and is perpendicular to the graph of $3x + 6y = 2$.

41. Write an equation of the line that is perpendicular to the line through $(9, 10)$ and $(3, -2)$ and passes through the x-intercept of that line.

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*.

42. $y = -2x + 11$
$y + 2x = 23$

43. $3y = 2x + 14$
$2x - 3y = 2$

44. $y = -5x$
$y = 5x - 18$

45. GEOMETRY The diagonals of a square are segments that connect the opposite vertices. Determine the relationship between the diagonals $\overline{AC}$ and $\overline{BD}$ of square $ABCD$.
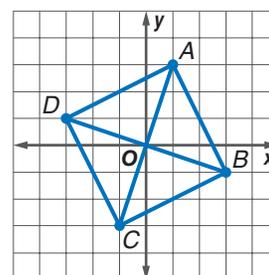
46. CRITICAL THINKING What is a if the lines with equations $y = ax + 5$ and $2y = (a + 4)x - 1$ are parallel?

CONTENTS

47. Answer the question that was posed at the beginning of the lesson.

How can you determine whether two lines are parallel?

Include the following in your answer:
- an equation whose graph is parallel to the graph of $y = -5x$, with an explanation of your reasoning, and
- an equation whose graph is perpendicular to the graph of $y = -5x$, with an explanation of your reasoning.

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

48. What is the slope of a line perpendicular to the graph of $3x + 4y = 24$?

Ⓐ $-\dfrac{4}{3}$ Ⓑ $-\dfrac{3}{4}$ Ⓒ $\dfrac{3}{4}$ Ⓓ $\dfrac{4}{3}$

49. How can the graph of $y = 3x + 4$ be used to graph $y = 3x + 2$?

Ⓐ Move the graph of the line right 2 units.
Ⓑ Change the slope of the graph from 4 to 2.
Ⓒ Change the y-intercept from 4 to 2.
Ⓓ Move the graph of the line left 2 units.

Maintain Your Skills

Mixed Review Write the point-slope form of an equation for a line that passes through each point with the given slope. *(Lesson 5-5)*

50. $(3, 5)$, $m = -2$ **51.** $(-4, 7)$, $m = 5$ **52.** $(-1, -3)$, $m = -\dfrac{1}{2}$

TELEPHONE For Exercises 53 and 54, use the following information.
An international calling plan charges a rate per minute plus a flat fee. A 10-minute call to the Czech Republic costs $3.19. A 15-minute call costs $4.29. *(Lesson 5-4)*

53. Write a linear equation in slope-intercept form to find the total cost C of an m-minute call.

54. Find the cost of a 12-minute call.

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Write the slope-intercept form of an equation of the line that passes through each pair of points. *(To review slope-intercept form, see Lesson 5-4.)*

55. $(5, -1)$, $(-3, 3)$ **56.** $(0, 2)$, $(8, 0)$ **57.** $(2, 1)$, $(3, -4)$

58. $(5, 5)$, $(8, -1)$ **59.** $(6, 9)$, $(4, 9)$ **60.** $(-6, 4)$, $(2, -2)$

Practice Quiz 2   Lessons 5-3 through 5-6

Write the slope-intercept form for an equation of the line that satisfies each condition.

1. slope 4 and y-intercept -3 *(Lesson 5-3)*

2. passes through $(1, -3)$ with slope 2   *(Lesson 5-4)*

3. passes through $(-1, -2)$ and $(1, 3)$ *(Lesson 5-4)*

4. parallel to the graph of $y = 2x - 2$ and passes through $(-2, 3)$ *(Lesson 5-6)*

5. Write $y - 4 = \dfrac{1}{2}(x + 3)$ in standard form and in slope-intercept form.   *(Lesson 5-5)*

CONTENTS

Statistics: Scatter Plots and Lines of Fit

What You'll Learn

- Interpret points on a scatter plot.
- Write equations for lines of fit.

Vocabulary

- scatter plot
- positive correlation
- negative correlation
- line of fit
- best-fit line
- linear interpolation

How do scatter plots help identify trends in data?

The points of a set of real-world data do not always lie on one line. But, you may be able to draw a line that seems to be close to all the points.

The line in the graph shows a linear relationship between the year x and the number of bushels of apples y. As the years increase, the number of bushels of apples also increases.

Apples in Storage in U.S.
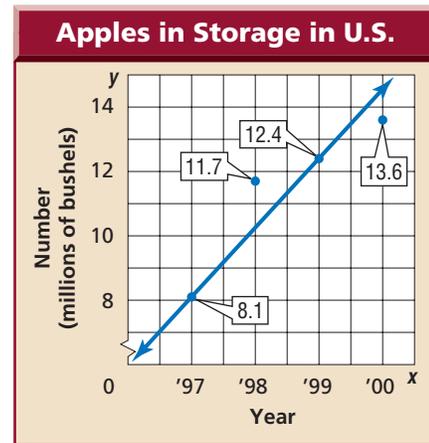
Source: U.S. Apple Association

INTERPRET POINTS ON A SCATTER PLOT A **scatter plot** is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane. Scatter plots are used to investigate a relationship between two quantities.

- In the first graph below, there is a **positive correlation** between x and y. That is, as x increases, y increases.
- In the second graph below, there is a **negative correlation** between x and y. That is, as x increases, y decreases.
- In the third graph below, there is *no correlation* between x and y. That is, x and y are not related.

If the pattern in a scatter plot is linear, you can draw a line to summarize the data. This can help identify trends in the data.
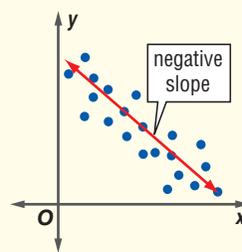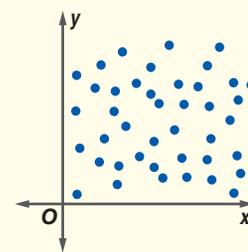
Key Concept — Scatter Plots

Positive Correlation

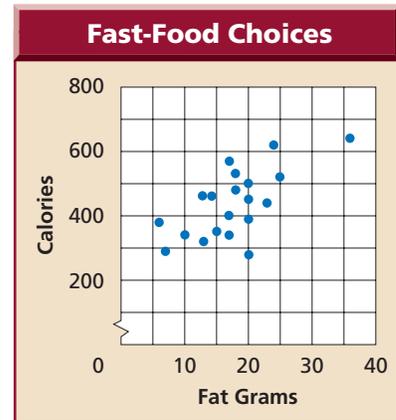Negative Correlation

No Correlation

CONTENTS
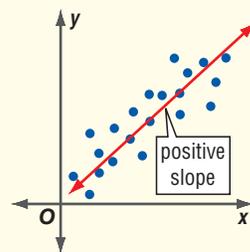
Example 1 Analyze Scatter Plots

Determine whether each graph shows a *positive correlation*, a *negative correlation*, or *no correlation*. If there is a positive or negative correlation, describe its meaning in the situation.

**a. NUTRITION** The graph shows fat grams and Calories for selected choices at a fast-food restaurant.

The graph shows a positive correlation. As the number of fat grams increases, the number of Calories increases.

Fast-Food Choices

Source: Olen Publishing Co.

b. CARS The graph shows the weight and the highway gas mileage of selected cars.

The graph shows a negative correlation. As the weight of the automobile increases, the gas mileage decreases.

Automobiles
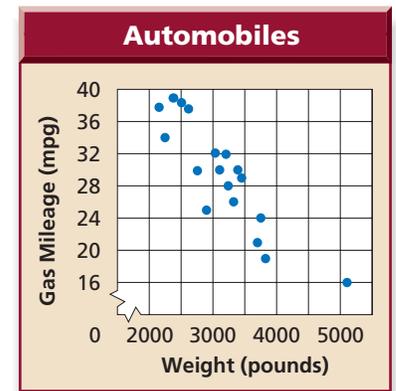
Source: Yahoo!

Is there a relationship between the length of a person's foot and his or her height? Make a scatter plot and then look for a pattern.

Algebra Activity

Making Predictions

Collect the Data

- Measure your partner's foot and height in centimeters. Then trade places.
- Add the points (foot length, height) to a class scatter plot.

Analyze the Data

1. Is there a correlation between foot length and height for the members of your class? If so, describe it.
2. Draw a line that summarizes the data and shows how the height changes as the foot length changes.

Make a Conjecture

3. Use the line to predict the height of a person whose foot length is 25 centimeters. Explain your method.

www.algebra1.com/extra_examples

CONTENTS

LINES OF FIT If the data points do not all lie on a line, but are close to a line, you can draw a **line of fit**. This line describes the trend of the data. Once you have a line of fit, you can find an equation of the line.

In this lesson, you will use a graphical method to find a line of fit. In the follow-up to Lesson 5-7, you will use a graphing calculator to find a line of fit. The calculator uses a statistical method to find the line that most closely approximates the data. This line is called the **best-fit line**.
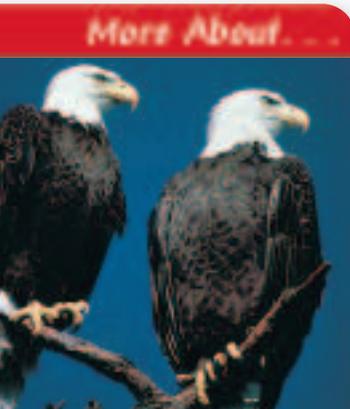
Example 2 Find a Line of Fit

BIRDS The table shows an estimate for the number of bald eagle pairs in the United States for certain years since 1985.

Years since 1985	3	5	7	9	11	14
Bald Eagle Pairs	2500	3000	3700	4500	5000	5800

Source: U.S. Fish and Wildlife Service

a. **Draw a scatter plot and determine what relationship exists, if any, in the data.**

Let the independent variable x be the number of years since 1985, and let the dependent variable y be the number of bald eagle pairs.

As the number of years increases, the number of bald eagle pairs increases. There is a positive correlation between the two variables.

Bald Eagle Pairs

b. **Draw a line of fit for the scatter plot.**

No one line will pass through all of the data points. Draw a line that passes close to the points. A line of fit is shown in the scatter plot at the right.

c. **Write the slope-intercept form of an equation for the line of fit.**

The line of fit shown above passes through the data points (3, 2500) and (11, 5000).

Step 1 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$m = \frac{5000 - 2500}{11 - 3} \qquad \text{Let } (x_1, y_1) = (3, 2500) \text{ and } (x_2, y_2) = (11, 5000).$$

$$m = \frac{2500}{8} \text{ or } 312.5 \qquad \text{Simplify.}$$

Step 2 Use $m = 312.5$ and either the point-slope form or the slope-intercept form to write the equation. You can use either data point. We chose (3, 2500).

Point-slope form	**Slope-intercept form**
$y - y_1 = m(x - x_1)$	$y = mx + b$
$y - 2500 = 312.5(x - 5)$	$2500 = 312.5(3) + b$
$y - 2500 = 312.5x - 937.5$	$2500 = 937.5 + b$
$y = 312.5x + 1562.5$	$1562.5 = b$
	$y = 312.5x + 1562.5$

Using either method, $y = 312.5x + 1562.5$.

CONTENTS

CHECK Check your result by substituting (11, 5000) into $y = 312.5x + 1562.5$.

$$y = 312.5x + 1562.5 \qquad \text{Line of fit equation}$$

$$5000 \stackrel{?}{=} 312.5(11) + 1562.5 \qquad \text{Replace } x \text{ with 11 and } y \text{ with 5000.}$$

$$5000 \stackrel{?}{=} 3437.5 + 1562.5 \qquad \text{Multiply.}$$

$$5000 = 5000 \checkmark \qquad \text{Add.}$$

The solution checks.

In Lesson 5-4, you learned about linear extrapolation, which is predicting values that are *outside* the range of the data. You can also use a linear equation to predict values that are *inside* the range of the data. This is called **linear interpolation**.

Example **3** *Linear Interpolation*

BIRDS Use the equation for the line of fit in Example 2 to estimate the number of bald eagle pairs in 1998.

Use the equation $y = 312.5x + 1562.5$, where x is the number of years since 1985 and y is the number of bald eagle pairs.

$$y = 312.5x + 1562.5 \qquad \text{Original equation}$$

$$y = 312.5(13) + 1562.5 \qquad \text{Replace } x \text{ with } 1998 - 1985 \text{ or 13.}$$

$$y = 5625 \qquad \text{Simplify.}$$

There were about 5625 bald eagle pairs in 1998.

Check for Understanding

Concept Check
1. **Explain** how to determine whether a scatter plot has a positive or negative correlation.

2. **OPEN ENDED** Sketch scatter plots that have each type of correlation.
   a. positive   **b.** negative   **c.** no correlation

3. **Compare and contrast** linear interpolation and linear extrapolation.

Guided Practice Determine whether each graph shows a *positive correlation*, a *negative correlation*, or *no correlation*. If there is a positive or negative correlation, describe its meaning in the situation.

4.
Test Scores

5.
Weekly Activities

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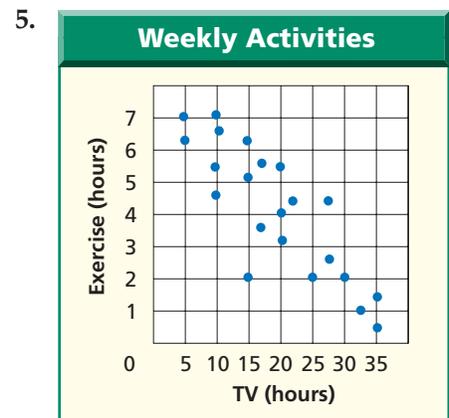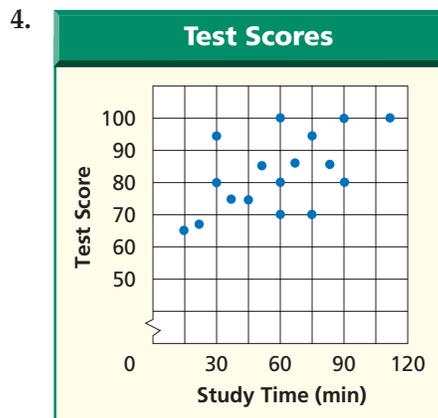
Application   **BIOLOGY**   For Exercises 6–9, use the table that shows the average body temperature in degrees Celsius of 9 insects at a given air temperature.

Temperature (°C)								
Air	25.7	30.4	28.7	31.2	31.5	26.2	30.1	31.5	18.2
Body	27.0	31.5	28.9	31.0	31.5	25.6	28.4	31.7	18.7

6. Draw a scatter plot and determine what relationship exists, if any, in the data.

7. Draw a line of fit for the scatter plot.

8. Write the slope-intercept form of an equation for the line of fit.

9. Predict the body temperature of an insect if the air temperature is 40.2°F.
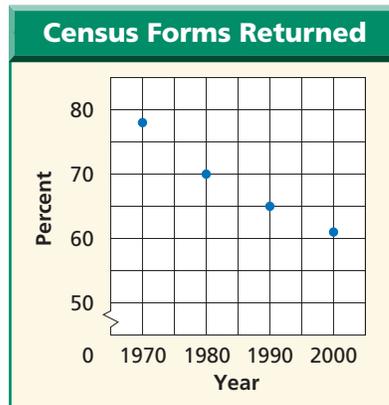
Practice and Apply

Determine whether each graph shows a *positive correlation*, a *negative correlation*, or *no correlation*. If there is a positive or negative correlation, describe its meaning in the situation.

10.

Census Forms Returned

Source: U.S. Census Bureau

11.

Hurricanes

Source: USA TODAY

12.

Electronic Tax Returns

Source: IRS

13.

Cereal Bars
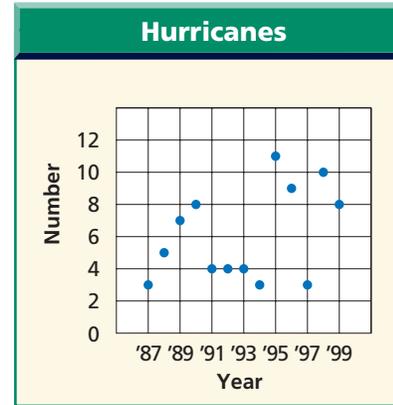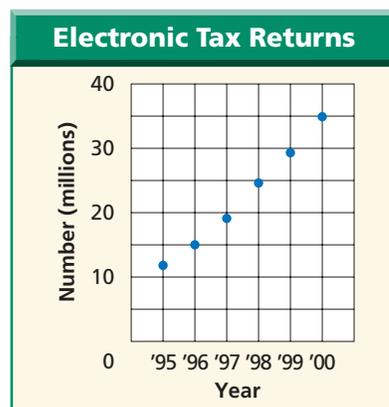
Source: Vitality

FARMING   For Exercises 14 and 15, refer to the graph at the top of page 298 about apple storage.

14. Use the points (1997, 8.1) and (1999, 12.4) to write the slope-intercept form of an equation for the line of fit.

15. Predict the number of bushels of apples in storage in 2002.

CONTENTS

USED CARS For Exercises 16 and 17, use the scatter plot that shows the ages and prices of used cars from classified ads.
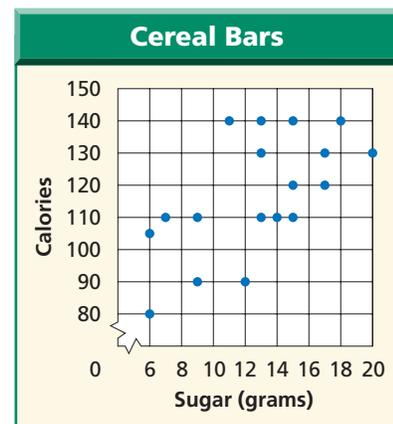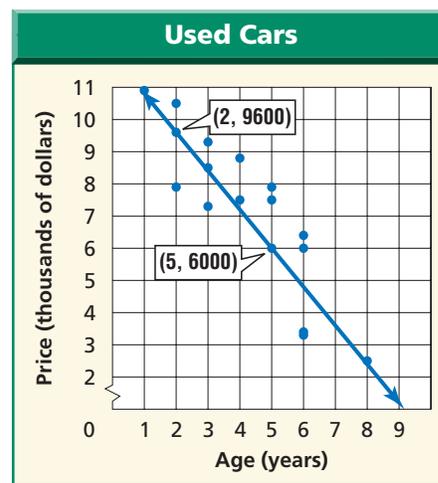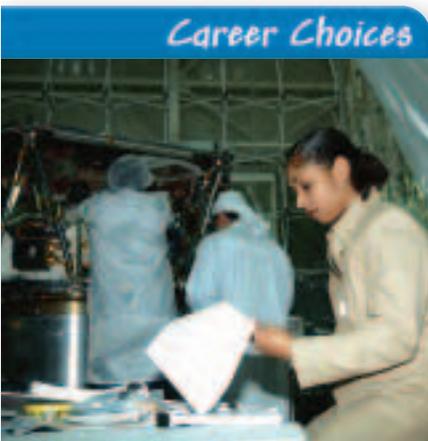
16. Use the points (2, 9600) and (5, 6000) to write the slope-intercept form of an equation for the line of fit shown in the scatter plot.

17. Predict the price of a car that is 7 years old.

Used Cars

Price (thousands of dollars) vs Age (years). Points labeled (2, 9600) and (5, 6000).

Source: *Columbus Dispatch*

PHYSICAL SCIENCE For Exercises 18–23, use the following information. Hydrocarbons are composed of only carbon and hydrogen atoms. The table gives the number of carbon atoms and the boiling points for several hydrocarbons.

18. Draw a scatter plot comparing the numbers of carbon atoms to the boiling points.

Hydrocarbons			
Name	**Formula**	**Number of Carbon Atoms**	**Boiling Point (°C)**
Ethane	C_2H_6	2	−89
Propane	C_3H_8	3	−42
Butane	C_4H_{10}	4	−1
Hexane	C_6H_{12}	6	69
Octane	C_8H_{18}	8	126

19. Draw a line of fit for the data.

20. Write the slope-intercept form of an equation for the line of fit.

21. Predict the boiling point for methane (CH_4), which has 1 carbon atom.

22. Predict the boiling point for pentane (C_5H_{12}), which has 5 carbon atoms.

23. The boiling point of heptane is 98.4°C. Use the equation of the line of fit to predict the number of carbon atoms in heptane.

SPACE For Exercises 24–28, use the table that shows the amount the United States government has spent on space and other technologies in selected years.

Federal Spending on Space and Other Technologies								
Year	1980	1985	1990	1995	1996	1997	1998	1999
Spending (billions of dollars)	4.5	6.6	11.6	12.6	12.7	13.1	12.9	12.4

Source: U.S. Office of Management and Budget

24. Draw a scatter plot and determine what relationship, if any, exists in the data.

25. Draw a line of fit for the scatter plot.

26. Let x represent the number of years since 1980. Let y represent the spending in billions of dollars. Write the slope-intercept form of the equation for the line of fit.

27. Predict the amount that will be spent on space and other technologies in 2005.

28. The government projects spending of $14.3 billion in space and other technologies in 2005. How does this compare to your prediction?

FORESTRY   For Exercises 29–33, use the table that shows the number of acres burned by wildfires in Florida each year and the corresponding number of inches of spring rainfall.

Florida's Burned Acreage and Spring Rainfall					
Year	Rainfall (inches)	Acres (thousands)	Year	Rainfall (inches)	Acres (thousands)
1988	17.5	194	1994	18.1	180
1989	12.0	645	1995	16.3	46
1990	14.0	250	1996	20.4	94
1991	30.1	87	1997	18.5	146
1992	16.0	83	1998	22.2	507
1993	19.6	80	1999	12.7	340

Source: Florida Division of Forestry

29. Draw a scatter plot with rainfall on the x-axis and acres on the y-axis.

30. Draw a line of fit for the data.

31. Write the slope-intercept form of an equation for the line of fit.

32. In 2000, there was only 8.25 inches of spring rainfall. Estimate the number of acres burned by wildfires in 2000.

33. In 1998, there was 22.2 inches of rainfall, yet 507,000 acres were burned. Where was this data graphed in the scatter plot? How did this affect the line of fit?

Online Research   **Data Update** What has happened to the number of acres burned by wildfires in Florida since 1999? Visit www.algebra1.com/data_update to learn more.

34. **CRITICAL THINKING**   A test contains 20 true-false questions. Draw a scatter plot that shows the relationship between the number of correct answers x and the number of incorrect answers y.

RESEARCH   For Exercises 35 and 36, choose a topic to research that you believe may be correlated, such as arm span and height. Find existing data or collect your own.

35. Draw a line of fit for the data.

36. Use the line to make a prediction about the data.

37. **WRITING IN MATH**   Answer the question that was posed at the beginning of the lesson.

How do scatter plots help identify trends in data?

Include the following in your answer:
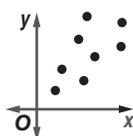- a scatter plot that shows a person's height and his or her age, with a description of any trends, and
- an explanation of how you could use the scatter plot to predict a person's age given his or her height.
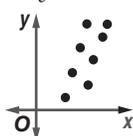
Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

38. Which graph is the best example of data that show a negative linear relationship between the variables x and y?
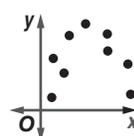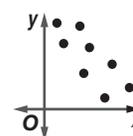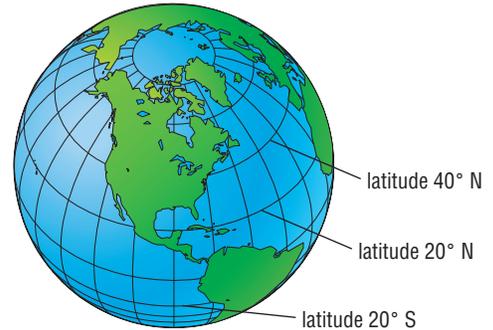
Ⓐ
Ⓑ
Ⓒ
Ⓓ

39. Choose the equation for the line that best fits the data in the table at the right.

(A) $y = x + 4$

(B) $y = 2x + 3$

(C) $y = 7$

(D) $y = 4x - 5$

x	y
1	5
2	7
3	7
4	11

Extending the Lesson

GEOGRAPHY   For Exercises 40–44, use the following information.
The *latitude* of a place on Earth is the measure of its distance from the equator.

40. MAKE A CONJECTURE   What do you think is the relationship between a city's latitude and its January temperature?

41. RESEARCH   Use the Internet or other reference to find the latitude of 15 cities in the northern hemisphere and the corresponding January mean temperatures.

latitude 40° N

latitude 20° N

latitude 20° S

42. Make a scatter plot and draw a line of fit for the data.

43. Write an equation for the line of fit.

44. MAKE A CONJECTURE   Find the latitude of your city and use the equation to predict its mean January temperature. Check your prediction by using another source such as the newspaper.

Maintain Your Skills

Mixed Review   **Write the slope-intercept form of an equation for the line that satisfies each condition.**   *(Lesson 5-6)*

45. parallel to the graph of $y = -4x + 5$ and passes through $(-2, 5)$

46. perpendicular to the graph of $y = 2x + 3$ and passes through $(0, 0)$

Write the point-slope form of an equation for a line that passes through each point with the given slope.   *(Lesson 5-5)*

47.

(−2, 3)

$m = -2$

48.

$m = 3$

(1, −2)

49.

$m = 1$

(−3, −3)

Find the x- and y-intercepts of the graph of each equation.   *(Lesson 4-5)*

50. $3x + 4y = 12$

51. $2x - 5y = 8$

52. $y = 3x + 6$

Solve each equation. Then check your solution.   *(Lesson 3-4)*

53. $\dfrac{r + 7}{-4} = \dfrac{r + 2}{6}$

54. $\dfrac{n - (-4)}{-3} = 7$

55. $\dfrac{2x - 1}{5} = \dfrac{4x - 5}{7}$

CONTENTS

Regression and Median-Fit Lines

One type of equation of best-fit you can find is a linear **regression equation**. Linear regression is sometimes called the *method of least squares*.

EARNINGS The table shows the average hourly earnings of U.S. production workers for selected years.

Year	1960	1965	1970	1975	1980	1985	1990	1995	1999
Earnings	$2.09	2.46	3.23	4.53	6.66	8.57	10.01	11.43	13.24

Source: Bureau of Labor Statistics

Find and graph a linear regression equation. Then predict the average hourly earnings in 2010.

Step 1 *Find a regression equation.*

- Enter the years in L1 and the earnings in L2.
  KEYSTROKES: *Review entering a list on page 204.*

- Find the regression equation by selecting LinReg(*ax+b*) on the STAT CALC menu.
  KEYSTROKES: STAT ▶ 4 ENTER

The equation is in the form $y = ax + b$.

```
LinReg
y=ax+b
a=.3006548524
b=-588.3498684
r²=.9769594099
r=.9884125707
```

The equation is about $y = 0.30x - 588.35$.

r is the **linear correlation coefficient**. The closer the absolute value of r is to 1, the better the equation models the data. Because the r value is close to 1, the model fits the data well.

Step 2 *Graph the regression equation.*

- Use STAT PLOT to graph the scatter plot.
  KEYSTROKES: *Review statistical plots on page 204.*

- Copy the equation to the Y= list and graph.
 **KEYSTROKES:** Y= VARS 5 ▶ ▶ 1 GRAPH
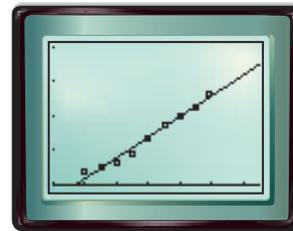
[1950, 2020] scl: 10 by [0, 20] scl: 5

The *residual* is the difference between actual and predicted data. The predicted earnings in 1970 using this model were $3.94. (To calculate, press 2nd [CALC] 1 1970 ENTER .) So the residual for 1970 was $3.94 – $3.23 or $0.71.

Step 3 *Predict using the regression equation.*

- Find y when $x = 2010$ using **value** on the CALC menu.
  KEYSTROKES: 2nd [CALC] 1 2010 ENTER

According to the regression equation, the average hourly earnings in 2010 will be about $15.97.

```
Y1=.30065485244904X+-58...



X=2010      Y=15.966385
```
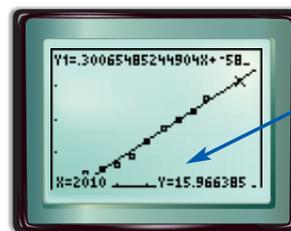
The graph and the coordinates of the point are shown.

CONTENTS

Investigation

A second type of best-fit line that can be found using a graphing calculator is a **median-fit line**. The equation of a median-fit line is calculated using the medians of the coordinates of the data points.

> **Find and graph a median-fit equation for the data on hourly earnings. Then predict the average hourly earnings in 2010. Compare this prediction to the one made using the regression equation.**
>
> **Step 1** *Find a median–fit equation.*
>
> - The data are already in Lists 1 and 2. Find the median-fit equation by using **Med-Med** on the **STAT CALC** menu.
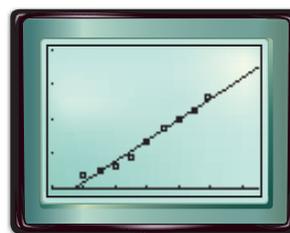>
>   **KEYSTROKES:** | STAT | | ▶ | 3 | ENTER |
>
>
> The median-fit equation is $y = 0.299x - 585.17$.
>
> **Step 2** *Graph the median–fit equation.*
>
> - Copy the equation to the Y= list and graph.
>
>   **KEYSTROKES:** | Y= | | CLEAR | | VARS | 5 ▶ ▶ 1 | GRAPH |
>
>
> [1950, 2010] scl: 10 by [0, 20] scl: 5
>
> **Step 3** *Predict using the median–fit equation.*
>
>   **KEYSTROKES:** | 2nd | [CALC] 1 2010 | ENTER |
>
> According to the median-fit equation, the average hourly earnings in 2010 will be about $15.82. This is slightly less than the predicted value found using the regression equation.
>
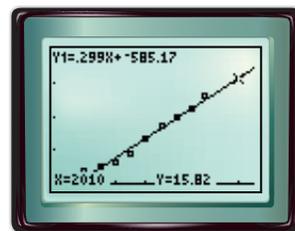
Exercises

Refer to the data on bald eagles in Example 2 on pages 300 and 301.

1. Find regression and median-fit equations for the data.

2. What is the correlation coefficient of the regression equation? What does it tell you about the data?

3. Use the regression and median-fit equations to predict the number of bald eagle pairs in 1998. Compare these to the number found in Example 3 on page 301.

For Exercises 4 and 5, use the table that shows the number of votes cast for the Democratic presidential candidate in selected North Carolina counties in the 1996 and 2000 elections.

4. Find regression and median-fit equations for the data.

5. In 1996, New Hanover County had 22,839 votes for the Democratic candidate. Use the regression and median-fit equations to estimate the number of votes for the Democratic candidate in that county in 2000. How do the predictions compare to the actual number of 29,292?

1996	2000
14,447	16,284
19,458	19,281
28,674	30,921
31,658	38,545
32,739	38,626
46,543	52,457
49,186	53,907
69,208	80,787
103,429	126,911
103,574	123,466

Source: NC State Board of Elections

www.algebra1.com/other_calculator_keystrokes

Vocabulary and Concept Check

best-fit line (p. 300)	line of fit (p. 300)	positive correlation (p. 298)
constant of variation (p. 264)	negative correlation (p. 298)	rate of change (p. 258)
direct variation (p. 264)	parallel lines (p. 292)	scatter plot (p. 298)
family of graphs (p. 265)	parent graph (p. 265)	slope (p. 256)
linear extrapolation (p. 283)	perpendicular lines (p. 293)	slope-intercept form (p. 272)
linear interpolation (p. 301)	point-slope form (p. 286)	

Exercises Choose the correct term to complete each sentence.

1. An equation of the form $y = kx$, where $k \neq 0$, describes a *(direct variation, linear extrapolation)*.

2. The ratio of *(rise, run)*, or vertical change, to the *(rise, run)*, or horizontal change, as you move from one point on a line to another, is the slope of a nonvertical line.

3. The lines with equations $y = -2x + 7$ and $y = -2x - 6$ are *(parallel, perpendicular)*.

4. The equation $y - 2 = -3(x - 1)$ is written in *(point-slope, slope-intercept)* form.

5. The equation $y = -\frac{1}{3}x + 6$ is written in *(slope-intercept, standard)* form.

6. The *(x-intercept, y-intercept)* of the equation $-x - 4y = 2$ is $-\frac{1}{2}$.
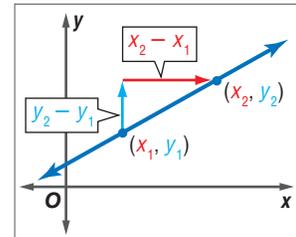
Lesson-by-Lesson Review

5-1 Slope

See pages 256–262.

Concept Summary

- The slope of a nonvertical line is the ratio of the rise to the run.
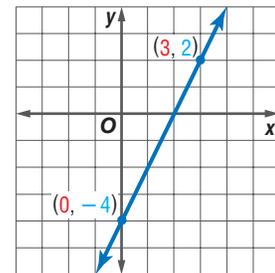- $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example Determine the slope of the line that passes through $(0, -4)$ and $(3, 2)$.

Let $(0, -4) = (x_1, y_1)$ and $(3, 2) = (x_2, y_2)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \dfrac{2 - (-4)}{3 - 0}$ $x_1 = 0, x_2 = 3, y_1 = -4, y_2 = 2$

$m = \dfrac{6}{3}$ or 2   Simplify.

Exercises Find the slope of the line that passes through each pair of points.
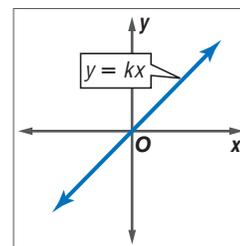*See Examples 1–4 on page 257.*

7. $(1, 3), (-2, -6)$ 8. $(0, 5), (6, 2)$ 9. $(-6, 4), (-6, -2)$

10. $(8, -3), (-2, -3)$ 11. $(2.9, 4.7), (0.5, 1.1)$ 12. $\left(\frac{1}{2}, 1\right), \left(-1, \frac{2}{3}\right)$

www.algebra1.com/vocabulary_review

CONTENTS

5-2 Slope and Direct Variation

See pages 264–270.

Concept Summary

- A direct variation is described by an equation of the form $y = kx$, where $k \neq 0$.
- In $y = kx$, k is the constant of variation. It is also the slope of the related graph.

Example Suppose y varies directly as x, and $y = -24$ when $x = 8$. Write a direct variation equation that relates x and y.

$$y = kx \qquad \text{Direct variation equation}$$
$$-24 = k(8) \qquad \text{Replace } y \text{ with } -24 \text{ and } x \text{ with } 8.$$
$$\frac{-24}{8} = \frac{k(8)}{8} \qquad \text{Divide each side by 8.}$$
$$-3 = k \qquad \text{Simplify.}$$

Therefore, $y = -3x$.

Exercises Graph each equation.   *See Examples 2 and 3 on page 265.*

13. $y = 2x$
14. $y = -4x$
15. $y = \frac{1}{3}x$
16. $y = -\frac{1}{4}x$
17. $y = \frac{3}{2}x$
18. $y = -\frac{4}{3}x$

Suppose y varies directly as x. Write a direct variation equation that relates x and y.   *See Example 4 on page 266.*

19. $y = -6$ when $x = 9$
20. $y = 15$ when $x = 2$
21. $y = 4$ when $x = -4$
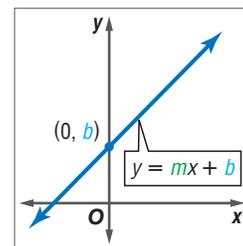**22.** $y = -6$ when $x = -18$
23. $y = -10$ when $x = 5$
24. $y = 7$ when $x = -14$

5-3 Slope-Intercept Form

See pages 272–277.

Concept Summary

- The linear equation $y = mx + b$ is written in slope-intercept form, where m is the slope, and b is the y-intercept.
- Slope-intercept form allows you to graph an equation quickly.
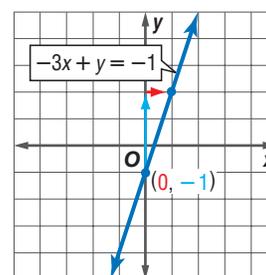

Example Graph $-3x + y = -1$.

$$-3x + y = -1 \qquad \text{Original equation}$$
$$-3x + y + 3x = -1 + 3x \qquad \text{Add 3x to each side.}$$
$$y = 3x - 1 \qquad \text{Simplify.}$$

Step 1 The y-intercept is -1. So, graph $(0, -1)$.

Step 2 The slope is 3 or $\frac{3}{1}$. From $(0, -1)$, move up 3 units and right 1 unit. Then draw a line.

Exercises Write an equation of the line with the given slope and *y*-intercept.
See Examples 1 and 2 on pages 272 and 273.

25. slope: 3, *y*-intercept: 2

26. slope: 1, *y*-intercept: −3

27. slope: 0, *y*-intercept: 4

28. slope: $\frac{1}{3}$, *y*-intercept: 2

29. slope: 0.5, *y*-intercept: −0.3

30. slope: −1.3, *y*-intercept: 0.4

Graph each equation. *See Examples 3 and 4 on pages 273 and 274.*

31. $y = 2x + 1$

32. $y = -x + 5$

33. $y = \frac{1}{2}x + 3$

34. $y = -\frac{4}{3}x - 1$

35. $5x - 3y = -3$

36. $6x + 2y = 9$

5-4 Writing Equations in Slope-Intercept Form

See pages 280–285.

Concept Summary

- To write an equation given the slope and one point, substitute the values of *m*, *x*, and *y* into the slope-intercept form and solve for *b*. Then, write the slope-intercept form using the values of *m* and *b*.
- To write an equation given two points, find the slope. Then follow the steps above.

Example Write an equation of a line that passes through
(−2, −3) with slope $\frac{1}{2}$.

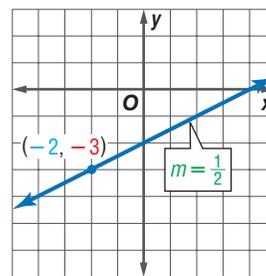$$y = mx + b \qquad \text{Slope-intercept form}$$

$$-3 = \frac{1}{2}(-2) + b \qquad \text{Replace } m \text{ with } \frac{1}{2}, y \text{ with } -3, \text{ and } x \text{ with } -2.$$

$$-3 = -1 + b \qquad \text{Multiply.}$$

$$-3 + 1 = -1 + b + 1 \qquad \text{Add 1 to each side.}$$

$$-2 = b \qquad \text{Simplify.}$$

Therefore, the equation is $y = \frac{1}{2}x - 2$.

Exercises Write an equation of the line that satisfies each condition.
See Examples 1 and 2 on pages 280 and 281.

37. passes through (−3, 3) with slope 1

38. passes through (0, 6) with slope −2

39. passes through (1, 6) with slope $\frac{1}{2}$

40. passes through (4, −3) with slope $-\frac{3}{5}$

41. passes through (−4, 2) and (1, 12)

42. passes through (5, 0) and (4, 5)

43. passes through (8, −1) with slope 0

44. passes through (4, 6) and has slope 0

CONTENTS

5-5 | Writing Equations in Point-Slope Form

See pages
286–291.

Concept Summary

- The linear equation $y - y_1 = m(x - x_1)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope.
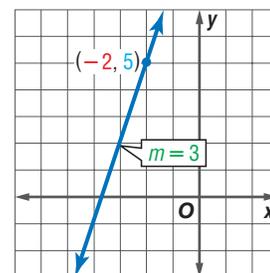
Example Write the point-slope form of an equation for a line that passes through $(-2, 5)$ with slope 3.

$y - y_1 = m(x - x_1)$ Use the point-slope form.

$y - 5 = 3[x - (-2)]$ $(x_1, y_1) = (-2, 5)$

$y - 5 = 3(x + 2)$ Subtract.

Exercises Write the point-slope form of an equation for a line that passes through each point with the given slope. *See Example 2 on page 287.*

45. $(4, 6)$, $m = 5$ **46.** $(-1, 4)$, $m = -2$ **47.** $(5, -3)$, $m = \frac{1}{2}$

48. $(1, -4)$, $m = -\frac{5}{2}$ **49.** $\left(\frac{1}{4}, -2\right)$, $m = 3$ **50.** $(4, -2)$, $m = 0$

Write each equation in standard form. *See Example 3 on page 287.*

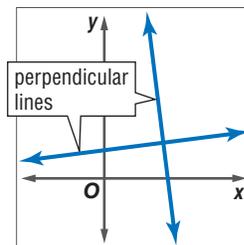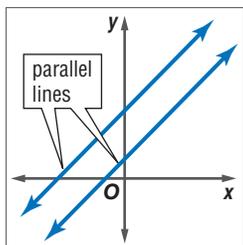**51.** $y - 1 = 2(x + 1)$ **52.** $y + 6 = \frac{1}{3}(x - 9)$ **53.** $y + 4 = 1.5(x - 4)$

5-6 | Geometry: Parallel and Perpendicular Lines

See pages
292–297.

Concept Summary

- Two nonvertical lines are parallel if they have the same slope.

- Two nonvertical lines are perpendicular if the product of their slopes is -1.

parallel lines

perpendicular lines

Example Write the slope-intercept form for an equation of the line that passes through $(5, -2)$ and is parallel to $y = 2x + 7$.

The line parallel to $y = 2x + 7$ has the same slope, 2.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-2) = 2(x - 5)$ Replace m with 2, y with -2, and x with 5.

$y + 2 = 2x - 10$ Simplify.

$y = 2x - 12$ Subtract 2 from each side.

CONTENTS

Chapter
5
For More ....
• Extra Practice, see pages 831–833.
• Mixed Problem Solving, see page 857.

Exercises Write the slope-intercept form for an equation of the line parallel to the graph of the given equation and passing through the given point. *See Example 1 on page 292.*

54. $y = 3x - 2$, $(4, 6)$ **55.** $y = -2x + 4$, $(6, -6)$ **56.** $y = -6x - 1$, $(1, 2)$

57. $y = \frac{5}{12}x + 2$, $(0, 4)$ **58.** $4x - y = 7$, $(2, -1)$ **59.** $3x + 9y = 1$, $(3, 0)$

Write the slope-intercept form for an equation of the line perpendicular to the graph of the given equation and passing through the given point. *See Example 3 on page 294.*

60. $y = 4x + 2$, $(1, 3)$ **61.** $y = -2x - 7$, $(0, -3)$ **62.** $y = 0.4x + 1$, $(2, -5)$

63. $2x - 7y = 1$, $(-4, 0)$ **64.** $8x - 3y = 7$, $(4, 5)$ **65.** $5y = -x + 1$, $(2, -5)$


**5-7** *Statistics: Scatter Plots and Lines of Fit*

See pages 298–305.

Concept Summary

• If y increases as x increases, then there is a positive correlation between x and y.

• If y decreases as x increases, then there is a negative correlation between x and y.

• If there is no relationship between x and y, then there is no correlation between x and y.

• A line of fit describes the trend of the data.

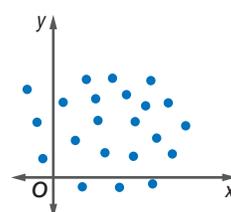• You can use the equation of a line of fit to make predictions about the data.
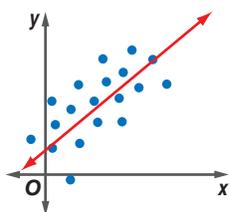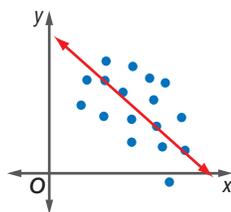
Positive Correlation   **Negative Correlation**   **No Correlation**

Exercises For Exercises 66–70, use the table that shows the length and weight of several humpback whales. *See Examples 2 and 3 on pages 300 and 301.*

Length (ft)	40	42	45	46	50	52	55
Weight (long tons)	25	29	34	35	43	45	51

66. Draw a scatter plot with length on the x-axis and weight on the y-axis.

67. Draw a line of fit for the data.

68. Write the slope-intercept form of an equation for the line of fit.

69. Predict the weight of a 48-foot humpback whale.

70. Most newborn humpback whales are about 12 feet in length. Use the equation of the line of fit to predict the weight of a newborn humpback whale. Do you think your prediction is accurate? Explain.

Vocabulary and Concepts

1. **Explain** why the equation of a vertical line cannot be in slope-intercept form.
2. **Draw** a scatter plot that shows a positive correlation.
3. **Name** the part of the slope-intercept form that represents the rate of change.

Skills and Applications

Find the slope of the line that passes through each pair of points.

4. $(5, 8), (-3, 7)$ 5. $(5, -2), (3, -2)$ 6. $(6, -3), (6, 4)$

7. **BUSINESS** A web design company advertises that it will design and maintain a website for your business for $9.95 per month. Write a direct variation equation to find the total cost C for any number of months m.

Graph each equation.

8. $y = 3x - 1$ 9. $y = 2x + 3$ 10. $2x + 3y = 9$

11. **WEATHER** The temperature is 16°F at midnight and is expected to fall 2° each hour during the night. Write the slope-intercept form of an equation to find the temperature T for any hour h after midnight.

Suppose y varies directly as x. Write a direct variation equation that relates x and y.

12. $y = 6$ when $x = 9$ 13. $y = -12$ when $x = 4$ 14. $y = -8$ when $x = 8$

Write the slope-intercept form of an equation of the line that satisfies each condition.

15. has slope -4 and y-intercept 3
16. passes through $(-2, -5)$ and $(8, -3)$
17. parallel to $3x + 7y = 4$ and passes through $(5, -2)$
18. a horizontal line passing through $(5, -8)$
19. perpendicular to the graph of $5x - 3y = 9$ and passes through the origin

20. Write the point-slope form of an equation for a line that passes through $(-4, 3)$ with slope -2.

ANIMALS For Exercises 21–24, use the table that shows the relationship between dog years and human years.

Dog Years	1	2	3	4	5	6	7
Human Years	15	24	28	32	37	42	47

21. Draw a scatter plot and determine what relationship, if any, exists in the data.
22. Draw a line of fit for the scatter plot.
23. Write the slope-intercept form of an equation for the line of fit.
24. Determine how many human years are comparable to 13 dog years.

25. **STANDARDIZED TEST PRACTICE** A line passes through $(0, 4)$ and $(3, 0)$. Which equation does *not* represent the equation of this line?

Ⓐ $y - 4 = -\frac{4}{3}(x - 0)$ Ⓑ $y = -\frac{4}{3}x + 3$ Ⓒ $\frac{x}{3} + \frac{y}{4} = 1$

Ⓓ $y - 0 = -\frac{4}{3}(x - 3)$ Ⓔ $4x + 3y = 12$

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If a person's weekly salary is x and she saves y, what fraction of her weekly salary does she spend?   (Lesson 1-1)

(A) $\dfrac{x}{y}$

(B) $\dfrac{x-y}{x}$

(C) $\dfrac{x-y}{y}$

(D) $\dfrac{y-x}{x}$

2. Evaluate $-2x + 7y$ if $x = -5$ and $y = 4$. (Lesson 2-6)

(A) 38

(B) 43

(C) 227

(D) 243

3. Find x, if $5x + 6 = 10$.   (Lesson 3-3)

(A) $-\dfrac{5}{4}$

(B) $\dfrac{1}{10}$

(C) $\dfrac{5}{16}$

(D) $\dfrac{4}{5}$

4. According to the data in the table, which of the following statements is true?   (Lesson 3-7)

Age	Frequency
8	1
10	3
14	2
16	1
17	2

(A) mean age = median age

(B) mean age > median age

(C) mean age < median age

(D) median age < mode age

5. What relationship exists between the x- and y-coordinates of each of the data points shown in the table?   (Lesson 4-8)

x	y
−3	4
−2	3
0	1
1	0
3	−2
5	−4

(A) x and y are opposites.

(B) The sum of x and y is 2.

(C) The y-coordinate is 1 more than the square of the x-coordinate.

(D) The y-coordinate is 1 more than the opposite of the x-coordinate.

6. What is the y-intercept of the line with equation $\dfrac{x}{3} - \dfrac{y}{2} = 1$?   (Lesson 4-5)

(A) −3

(B) −2

(C) $\dfrac{2}{3}$

(D) $\dfrac{3}{2}$

7. Find the slope of a line that passes through $(2, 4)$ and $(-4, 7)$.   (Lesson 5-1)

(A) $-\dfrac{1}{2}$

(B) $\dfrac{1}{2}$

(C) −2

(D) 2

8. Which equation represents the line that passes through $(3, 7)$ and $(10, 21)$?   (Lesson 5-4)

(A) $x + y = 10$

(B) $y = \dfrac{1}{2}x + \dfrac{11}{2}$

(C) $y = 2x + 1$

(D) $y = 3x - 2$

9. Choose the equation of a line parallel to the graph of $y = 3x + 4$.   (Lesson 5-6)

(A) $y = -\dfrac{1}{3}x + 4$

(B) $y = -3x + 4$

(C) $y = -x + 1$

(D) $y = 3x + 5$

CONTENTS

Preparing for Standardized Tests
For test-taking strategies and more
practice, see pages 867–884.

Part 2 Short Response/Grid In

Record your answers on the answer sheet
provided by your teacher or on a sheet of
paper.

10. While playing a game with her friends,
Ellen scored 12 points less than twice the
lowest score. She scored 98. What was the
lowest score in the game?   **(Lesson 3-4)**

11. Find the x- and y-intercepts of $3x - 2y = 12$.
(Lesson 4-5)

12. Find the slope of a line that passes through
$(5, 2)$ and $(-7, -3)$.   **(Lesson 5-1)**

13. Suppose y varies directly as x, and $y = -9$
when $x = 3$. Find x when $y = 18$.   **(Lesson 5-2)**

14. The graph of
$3x + 2y = 3$ is
shown at the right.
What is the
y-intercept?
(Lesson 5-3)
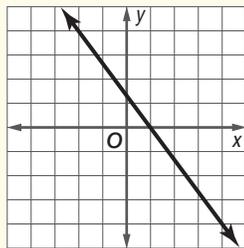
15. The table of ordered pairs
shows the coordinates of
some of the points on the
graph of a function.

What is the y-coordinate
of a point $(5, y)$ that lies on
the graph of the function?
(Lesson 5-4)

| x	y
−1	6
0	4
1	2
2	0
3	−2

16. The equation $y - 3 = -2(x + 5)$ is written
in point-slope form. What is the slope of
the line?   **(Lesson 5-5)**

17. Find the slope-intercept form of an
equation that passes through $(5, -8)$ and
is perpendicular to the graph of
$-5x + 2y = 14$.   **(Lesson 5-6)**

Part 3 Extended Response

Record your answers on a sheet of paper.
Show your work.

18. A friend wants to enroll for cellular phone
service. Three different plans are available.
(Lesson 5-3)

Plan 1 charges $0.59 per minute.

Plan 2 charges a monthly fee of $10, plus
$0.39 per minute.

Plan 3 charges a monthly fee of $59.95.

a. For each plan, write an equation that
represents the monthly cost C for m
number of minutes per month.

b. Graph each of the three equations.

c. Your friend expects to use 100 minutes
per month. In which plan do you think
that your friend should enroll? Explain.

19. A record company
is designing a new
CD case in the shape
of a parallelogram.
(Lesson 5-5)
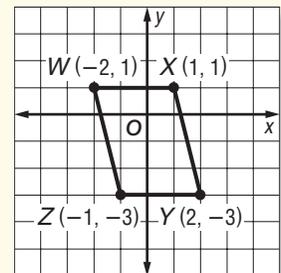
a. Find the
point-slope form
of the equation of
the line containing
side $\overline{WZ}$.

b. Find the standard form of the equation of
the line containing side $\overline{WZ}$.
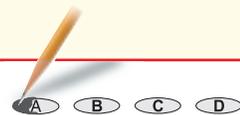
Test-Taking Tip Ⓐ Ⓑ Ⓒ Ⓓ
Questions 9 and 17
Recall that graphs of linear equations that are parallel
have the same slope. Graphs of linear equations that are
perpendicular have slopes that are opposite reciprocals of
each other.

www.algebra1.com/standardized_test

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